# Alevel Further Maths Induction Pack 2025

$22 + 7i$

$3 + 5i$

**Welcome!**

First of all, welcome to A level Further Maths! This pack is designed to support you as you prepare to study Further Maths.

A level Further Mathematics is an interesting and challenging course which broadens and deepens the topics you will learn at A level. It allows you to look at a wider range of topics such as matrices and complex numbers.

Mathematics qualifications are very well-respected by employers and are “facilitating” subjects for entry to higher education. Many universities encourage students to study Further maths if they are planning on pursuing a degree course in Mathematics, Physics or Engineering.

Students who have studied Further Mathematics often find the transition to university far more straightforward.

This induction pack includes tasks which form part of the assessment to judge your suitability to access the course in September. To access the course you need to complete these induction tasks thoroughly.

If you get stuck on any of the tasks, feel free to email
me on: michelle.cole@obamail.co.uk

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2},$$

Matrix A · Matrix B

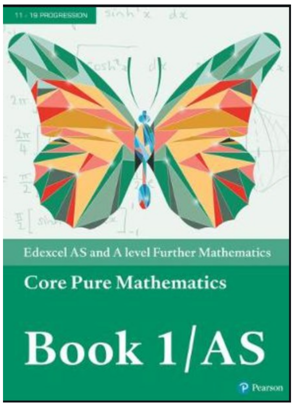
$$\begin{bmatrix} 1 & 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 5 & 8 \\ 7 & 9 \end{bmatrix}$$

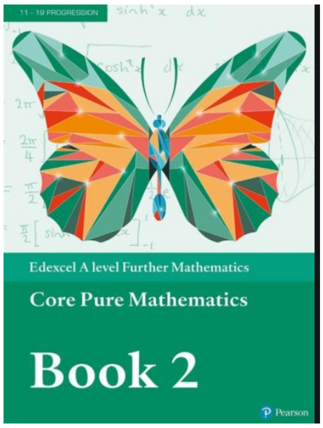
$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

# Key Information for A Level Further Maths

**Specification:** Pearson https://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2017/specification-and-sample-assesment/a-level-l3-further-mathematics-specification.pdf

**Textbooks–** will be provided by the school




Core Pure– 50% of the content.

**Content overview**

Proof, Complex numbers, Matrices, Further Algebra and functions, Further calculus, Further vectors, Polar coordinates, Hyperbolic functions, Differential equations

Further maths options—50% of the content—to be decided.

| Column A | Column B |
|---|---|
| Further Pure Mathematics 1 | Further Pure Mathematics 2 |
| Further Statistics 1 | Further Statistics 2 |
| Further Mechanics 1 | Further Mechanics 2 |
| Decision Mathematics 1 | Decision Mathematics 2 |

**A level Further Mathematics**

<table>
<tr><td>Paper 1: Core Pure Mathematics 1<br>25%<br>1 hour 30 mins<br>75 marks</td><td rowspan="2">Compulsory content – any content can be assessed on either paper</td></tr>
<tr><td>Paper 2: Core Pure Mathematics 2<br>25%<br>1 hour 30 mins<br>75 marks</td></tr>
<tr><td>Paper 3: Further Mathematics Option 1<br>25%<br>1 hour 30 mins<br>75 marks</td><td rowspan="2">Students take two optional papers with options available in<br>• Further Pure Mathematics<br>• Further Statistics<br>• Further Mechanics<br>• Decision Mathematics<br><br>See below for details of how these options can be arranged</td></tr>
<tr><td>Paper 4: Further Mathematics Option 2<br>25%<br>1 hour 30 mins<br>75 marks</td></tr>
</table>

## A Level Further Maths Preparatory Tasks

You should work through each of the  tasks shared in this booklet, ensuring that you are confident with all of the skills.

Please ensure that you keep all of your work demonstrating full working out to bring to the first session in September.

The solutions to all tasks are shared at the end of this booklet. You should self check your work.

### The ‘ careful reading ‘ Introductory task

Follow the instructions on page 14 to complete the reading task. If you found this article interesting then you can find plenty more at:

### Mathematical Problem Solving

A selection of mathematical puzzles aimed at transition into A Level for you to have a go at. Links to solutions shared.

### Just for fun- Additional suggested reading if you are interested in Maths

Some suggested reading if you want to further explore the beauty of maths

Matrix A · Matrix B

$$\begin{bmatrix} 1 & 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 5 & 8 \\ 7 & 9 \end{bmatrix}$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

$$22 + 7i$$

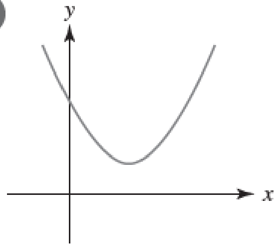
## Task 1 The Discriminant

Inside the square root of the quadratic formula you have the expression $b^2 - 4ac$. This expression is called the **discriminant**. You can use the discriminant to determine how many roots the equation has.

**Key point**

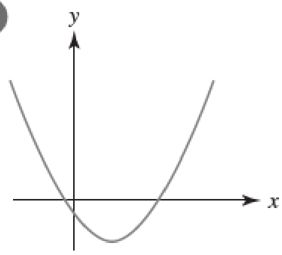
1. If $b^2 - 4ac < 0$ then the equation has no real roots.
2. If $b^2 - 4ac > 0$ then the equation has two real roots.
3. If $b^2 - 4ac = 0$ then the equation has one real root.

1



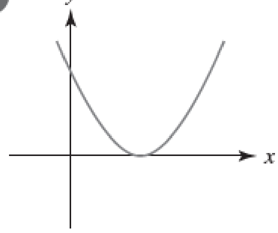

The curve does not cross the $x$-axis so the discriminant is negative.

2




The curve crosses the $x$-axis twice so the discriminant is positive.

3




The curve touches the $x$-axis once so the discriminant equals zero.

**Example 2**

Given that the quadratic equation $x^2 + 3x + k + 1 = 0$ has exactly one solution, find the value of $k$

$a = 1, b = 3, c = k + 1$

So $b^2 - 4ac = 3^2 - 4 \times 1 \times (k+1)$ — Find the discriminant.

$= 5 - 4k$

$5 - 4k = 0 \Rightarrow k = \frac{5}{4}$ — The equation has exactly one solution so the discriminant is zero.

**Example 3**

Given that the quadratic equation $5x^2 + 3x - k = 0$ has real solutions, find the range of possible values of $k$

$a = 5, b = 3, c = -k$

So $b^2 - 4ac = 5^2 - 4 \times 5 \times (-k)$ — Find the discriminant.

$= 25 + 20k$

$25 + 20k \geq 0 \Rightarrow k \geq -\frac{5}{4}$ — The equation has real solutions so the discriminant is greater than or equal to zero.

**Example 4**

Given that the quadratic equation $-x^2+7x+3-k=0$ has no real solutions, find the range of possible values of $k$

$a=-1,\ b=7,\ c=3-k$

So $b^2-4ac=7^2-4\times(-1)\times(3-k)$ — Find the discriminant.

$=61-4k$

$61-4k<0 \Rightarrow k>\frac{61}{4}$ — The equation has no solutions so the discriminant is negative.

**Exercise**

1) Work out how many real solutions each of these quadratic equations has.

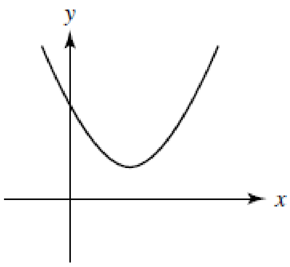
**a** $x^2-5x+7=0$

**b** $7-2x-3x^2=0$

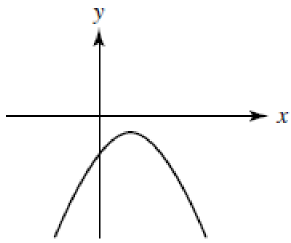
**c** $4x^2-28x+49=0$ .

2) Choose a possible equation from the box for each of the graphs.

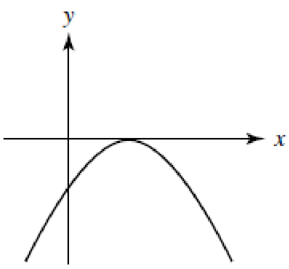
**a**



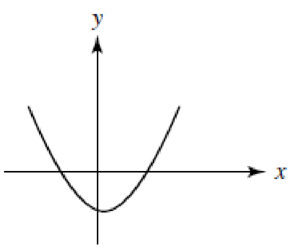

**d**




**b**




**c**




| |
|---|
| $y=-4x^2+12x-9$ |
| $y=-x^2+2x-4$ |
| $y=7x^2-5x+4$ |
| $y=-x^2+x+6$ |
| $y=6x^2-x-15$ |

3)

Find the value of $k$ in each equation given that they each have exactly one solution.

**a** $3x^2+2x-k=0$

**b** $kx^2-x+4=0$

**c** $2x^2+5x+k-5=0$

4) Find the range of possible values of $k$ for each equation given that they all have real solutions.

**a** $x^2+3x-3k=0$

**b** $kx^2-7x+4=0$

**c** $-x^2+6x-k-2=0$

5)

Find the range of possible values of $k$ for each equation given that they all have no real solutions.

**a** $5x^2-x+2k=0$

**b** $-kx^2+4x+5=0$

**c** $6x^2-5x+3-2k=0$

## Task 2: Circles

$$(x - a)^2 + (y - b)^2 = r^2$$

# This is the general equation of a circle.

**Section A**

Write down an equation of the circle with the given centre and radius in each case.

**a** centre (0, 0) radius 5 **b** centre (1, 3) radius 2 **c** centre (4, –6) radius 1

**d** centre (–1, –8) radius 3 **e** centre $(-\frac{1}{2}, \frac{1}{2})$ radius $\frac{1}{2}$ **f** centre (–3, 9) radius $2\sqrt{3}$

Write down the coordinates of the centre and the radius of each of the following circles.

**a** $x^2 + y^2 = 16$ **b** $(x - 6)^2 + (y - 1)^2 = 81$ **c** $(x + 1)^2 + (y - 4)^2 = 121$

**d** $(x - 7)^2 + y^2 = 0.09$ **e** $(x + 2)^2 + (y + 5)^2 = 32$ **f** $(x - 8)^2 + (y + 9)^2 = 108$

Find the centre and radius of the circle with equation:

$$x^2 + y^2 - 14x + 16y - 12 = 0$$

$$x^2 + y^2 - 14x + 16y - 12 = 0$$

Rearrange to have the x and y terms together

$$x^2 - 14x + y^2 + 16y - 12 = 0$$

Complete the square for the x and y terms separately

$$(x - 7)^2 - (49) + (y + 8)^2 - (64) - 12 = 0$$

Group the like terms

$$(x - 7)^2 + (y + 8)^2 - 125 = 0$$

Add 125

$$(x - 7)^2 + (y + 8)^2 = 125$$

$$Centre = (7, -8)$$

$$Radius = \sqrt{125} = 5\sqrt{5}$$

**Section B**

Find the coordinates of the centre and the radius of each of the following circles.

**a** $x^2 + y^2 - 4y + 3 = 0$

**b** $x^2 + y^2 - 2x - 10y - 23 = 0$

**c** $x^2 + y^2 + 12x - 8y + 36 = 0$

**d** $x^2 + y^2 - 2x + 16y = 35$

**e** $x^2 + y^2 = 8x - 6y$

**f** $x^2 + y^2 + 10x - 2y - 19 = 0$

**g** $4x^2 + 4y^2 - 4x - 24y + 1 = 0$

**h** $9x^2 + 9y^2 + 6x - 24y + 8 = 0$

# Intersecting Circles and Straight Lines:

You can use algebra to find the coordinates of intersection between straight lines and circles.

Find the coordinates where the line y = x + 5 intersects the circle

$x^2 + (y - 2)^2 = 29.$

This is effectively solving simultaneous equations, where one is a quadratic (although actually it is a circle). You can solve by substitution. Replace the y in the circle equation with x + 5 since we are told these are equivalent...

$$x^2 + (y - 2)^2 = 29$$

Replace y with x + 5

$$x^2 + (x + 5 - 2)^2 = 29$$

Simplify the bracket

$$x^2 + (x + 3)^2 = 29$$

Expand the squared bracket

$$x^2 + x^2 + 6x + 9 = 29$$

Group terms on the left side

$$2x^2 + 6x - 20 = 0$$

Divide by 2

$$x^2 + 3x - 10 = 0$$

Factorise

$$(x + 5)(x - 2) = 0$$

Find solutions

$$x = -5 \ or \ x = 2$$

**Section C**

Find in each case the coordinates of the points where the line $l$ intersects the circle $C$.

**a** $l : y = x - 4$ $\quad C : x^2 + y^2 = 10$

**b** $l : 3x + y = 17$ $\quad C : x^2 + y^2 - 4x - 2y - 15 = 0$

**c** $l : y = 2x + 2$ $\quad C : 4x^2 + 4y^2 + 4x - 8y - 15 = 0$

The line with equation $y = 1 - x$ intersects the circle with equation $x^2 + y^2 + 6x + 2y = 27$ at the points $A$ and $B$.

Find the length of the chord $AB$, giving your answer in the form $k\sqrt{2}$ .

**Task 3 : Factor theorem and Polynomial division**

# The Factor Theorem:

The factor theorem is a quick way of finding simple linear factors of a polynomial

The factor theorem states that, if $f(x)$ is a polynomial:

If $f(p) = 0$, then $(x - p)$ is a factor of $f(x)$

If $f(3) = 0$, then $(x - 3)$ is a factor of $f(x)$

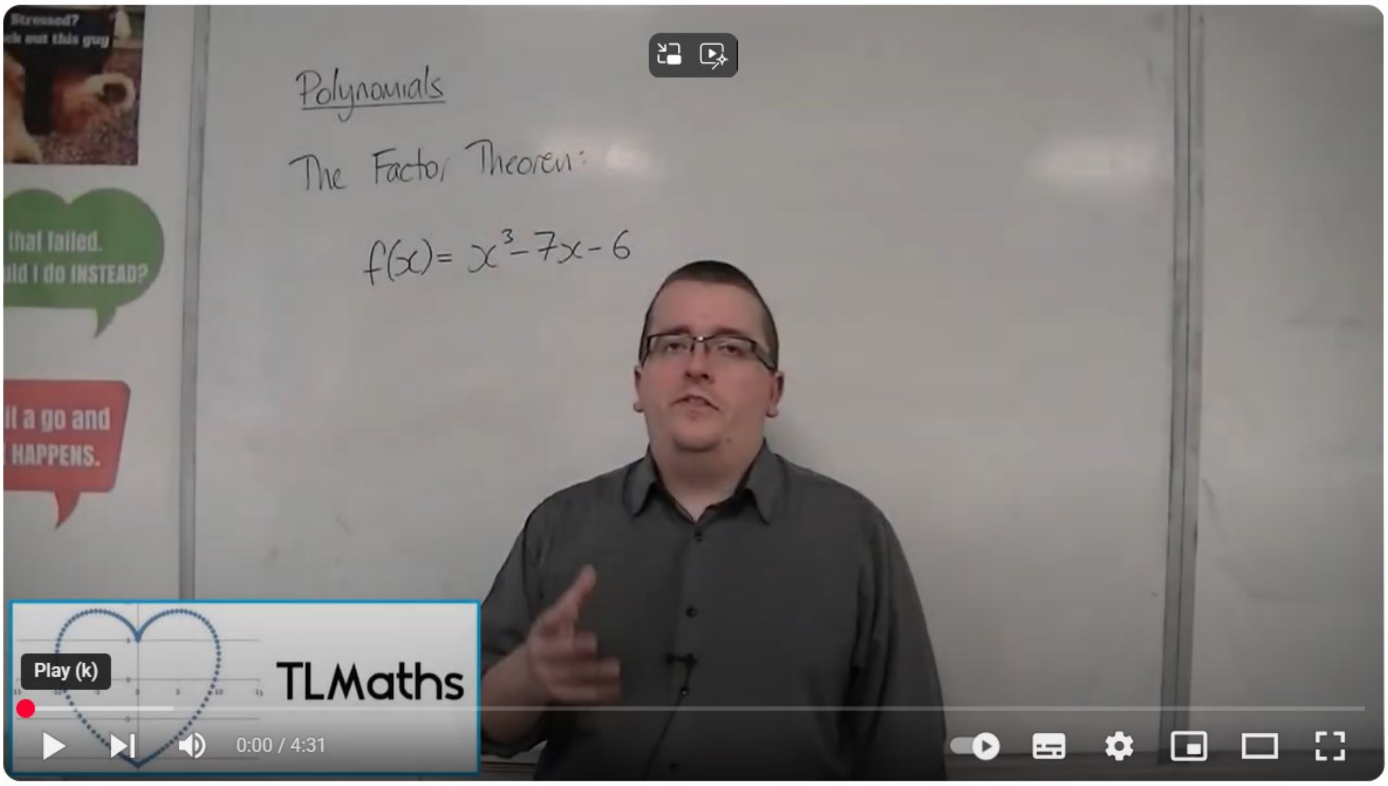
If $f(-1) = 0$, then $(x + 1)$ is a factor of $f(x)$

If $(x - p)$ is a factor of $f(x)$, then $f(p) = 0$

If $(x - 5)$ is a factor of $f(x)$, then $f(5) = 0$

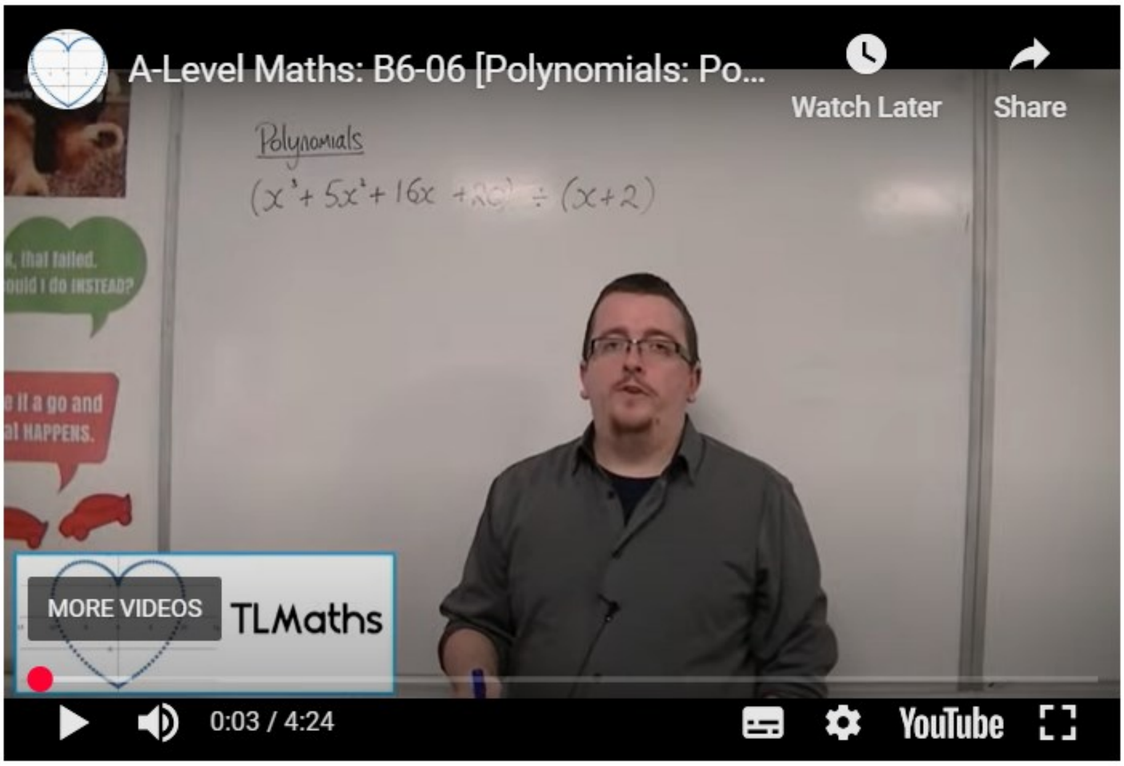
If $(x + 4)$ is a factor of $f(x)$, then $f(-4) = 0$

If you need further support understanding the factor theorem watch the video referenced below.




**A-Level Maths: B6-11 [Polynomials: Introducing the Factor Theorem]**

If you need

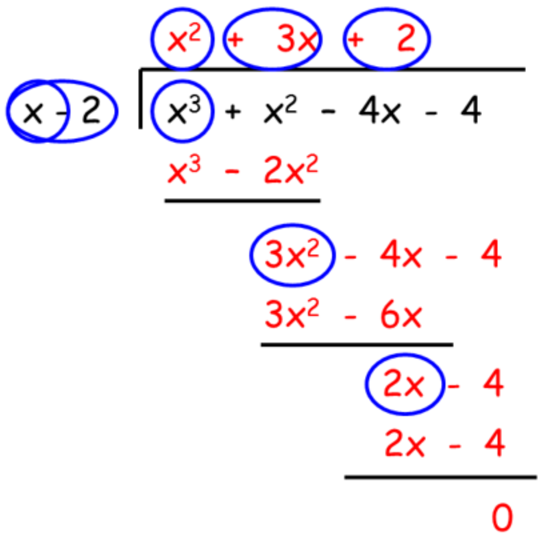



**B6-06 [Polynomials: Polynomial Division using the Long Division Method]**

# Example:

Show that $(x-2)$ is a factor of $x^3+x^2-4x-4$ by:

a) Algebraic division




The remainder is 0, so $(x-2)$ is a factor!

b) The factor theorem

$$x^3+x^2-4x-4$$

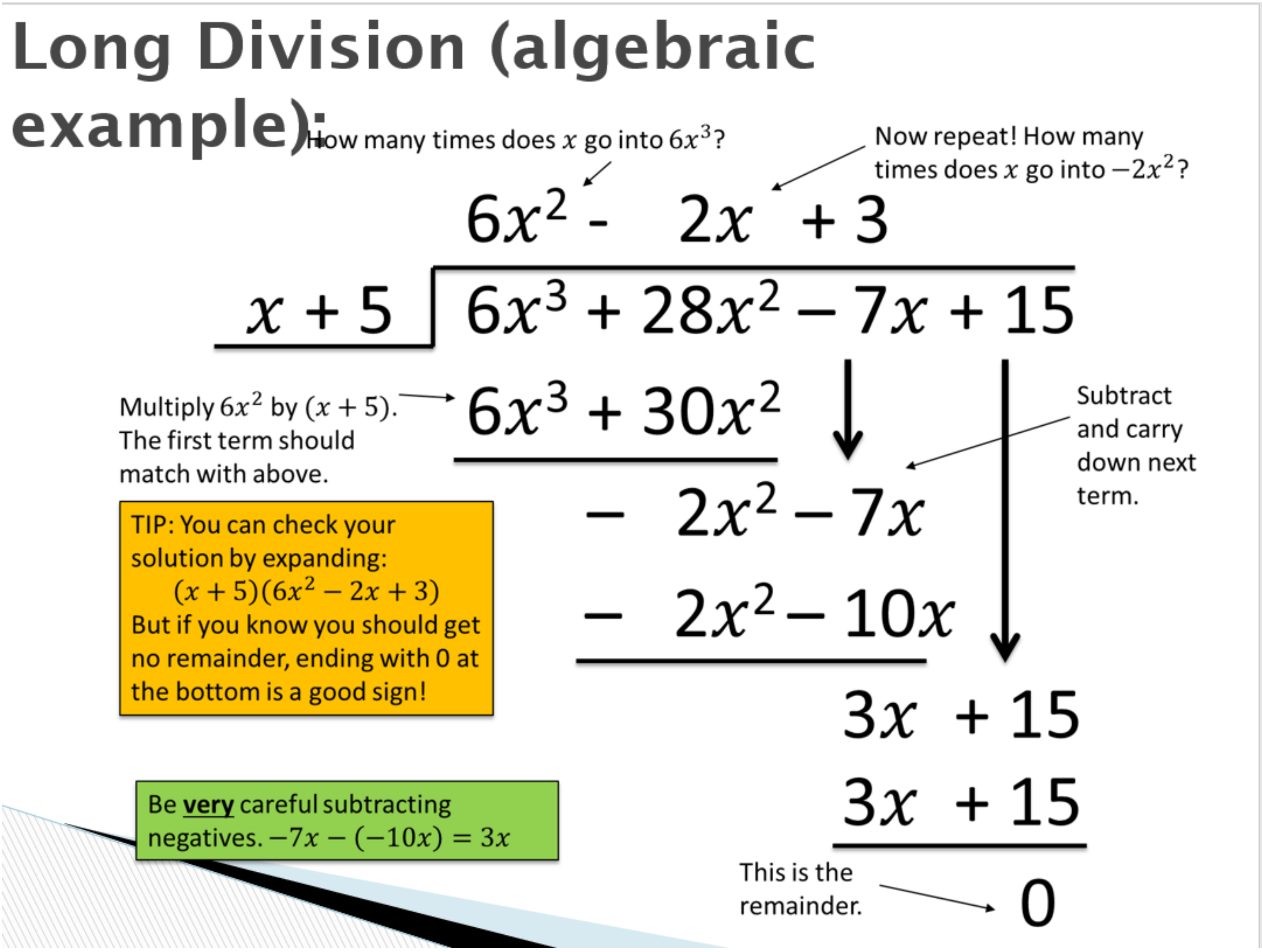
Sub in x = 2

$$(2)^3+(2)^2-4(2)-4$$

Calculate

$$=0$$

The answer is 0, so $(x-2)$ is a factor!




**Section A**

Use the factor theorem to determine whether or not

**a** $(x - 1)$ is a factor of $(x^3 + 2x^2 - 2x - 1)$

**b** $(x + 2)$ is a factor of $(x^3 - 5x^2 - 9x + 2)$

**c** $(x - 3)$ is a factor of $(x^3 - x^2 - 14x + 27)$

**d** $(x + 6)$ is a factor of $(2x^3 + 13x^2 + 2x - 24)$

**e** $(2x + 1)$ is a factor of $(2x^3 - 5x^2 + 7x - 14)$

**f** $(3x - 2)$ is a factor of $(2 - 17x + 25x^2 - 6x^3)$

$$f(x) \equiv x^3 - 2x^2 - 11x + 12.$$

**a** Show that $(x - 1)$ is a factor of $f(x)$.

**b** Hence, express $f(x)$ as the product of three linear factors.

$$g(x) \equiv 2x^3 + x^2 - 13x + 6.$$

Show that $(x + 3)$ is a factor of $g(x)$ and solve the equation $g(x) = 0$.

**Section B**

Find the quotient obtained in dividing

**a** $(x^3 + 2x^2 - x - 2)$ by $(x + 1)$

**b** $(x^3 + 2x^2 - 9x + 2)$ by $(x - 2)$

**c** $(20 + x + 3x^2 + x^3)$ by $(x + 4)$

**d** $(2x^3 - x^2 - 4x + 3)$ by $(x - 1)$

**e** $(6x^3 - 19x^2 - 73x + 90)$ by $(x - 5)$

**f** $(-x^3 + 5x^2 + 10x - 8)$ by $(x + 2)$

**g** $(x^3 - 2x + 21)$ by $(x + 3)$

**h** $(3x^3 + 16x^2 + 72)$ by $(x + 6)$

## Task 4 : Radians

### Using Radians

To convert between radians and degrees, you can use the fact that:

- $1\ radian = \frac{180°}{\pi}$

It helps to remember the following angles in radians:

- $30° = \frac{\pi}{6}$
- $90° = \frac{\pi}{2}$
- $45° = \frac{\pi}{4}$
- $180° = \pi$
- $60° = \frac{\pi}{3}$
- $360° = 2\pi$

**Degrees to radians:** $\div 180, \times \pi$
**Radians to degrees:** $\div \pi, \times 180$

**1** Convert each angle from degrees to radians, giving your answers in terms of π.

**a** 180° **b** 30° **c** 45° **d** 720° **e** 18° **f** 120°
**g** 15° **h** 40° **i** 270° **j** 7.5° **k** 144° **l** 220°

**2** Convert each angle from degrees to radians, giving your answers to 2 decimal places.

**a** 10° **b** 38° **c** 291° **d** 63.8° **e** 507° **f** 126.2°

**3** Convert each angle from radians to degrees.

**a** $2\pi$ **b** $\frac{\pi}{3}$ **c** $\frac{\pi}{2}$ **d** $\frac{3\pi}{4}$ **e** $\frac{\pi}{18}$ **f** $\frac{\pi}{30}$
**g** $\frac{5\pi}{6}$ **h** $\frac{\pi}{8}$ **i** $3\pi$ **j** $\frac{2\pi}{15}$ **k** $\frac{7\pi}{3}$ **l** $\frac{9\pi}{20}$

**4** Convert each angle from radians to degrees, giving your answers to 1 decimal place.

**a** $2^c$ **b** $0.5^c$ **c** $3.1^c$ **d** $1.43^c$ **e** $8.7^c$ **f** $0.742^c$

Without a calculator, work out the value of each below.

$\tan(225°) =$

$\sin(-45°) =$

$\tan(210°) =$

$\cos(750°) =$

$\sin(150°) =$

$\cos(120°) =$

$\cos(300°) =$

## **The ‘ Careful Reading’ Introductory Task**

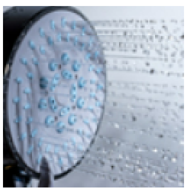
Careful reading is the skill of closely reading a text for deep understanding. Many students find this difficult, but it is definitely a skill you can learn.  You may ask yourself the relevance of this within A Level maths but with the increasing length of questions sometimes worth 10 to 12 marks full understanding of the scenario, information and question is vital.

When reading some text, ask yourself: how many times did I read the text? Did I reread the more difficult sections? Did I summarise the key information?

TASK: Follow the instructions in the grid and then complete the reading task that follows

| CAREFUL READING SKILL SHEET |
| --- |
| **1. READ FOR THE GIST**<br>Read the text twice to get the basic idea. Pay special attention to titles, diagrams and the first and last paragraphs. |
| **2. HELP I DON’T UNDERSTAND THIS BIT!**<br>Read the tricky scetions few times. Use a dictionary to check meanings. If you still don’t get it, mark the section with a question mark, and continue reading. Then go back to it when you have read the whole text. |
| **3. UNDERLINE THE MAIN POINTS:**<br>Try to underline about 10 per cent or less, 20 per cent maximum. Use a pencil so you can rub out underlinings. |
| **4. SUMMARISE WITH KEY POINTS OR A MIND-MAP OR OTHER SUITABLE GRAPHIC**<br>Read back over the underlined bits to ensure you have all the really important bits in your summary. |
| **NOW YOU WILL UNDERSTAND THE TEXT MUCH BETTER!** |

Read the article on the link below and then create a mind map of the key themes and findings of the article: https://plus.maths.org/content/shower-equation



## The shower equation: Dealing with delay

Chris Budd

# Power Quady

**Age 16 to 18**
**Challenge Level** ★

Find all real solutions of the equation

$$(x^2 - 7x + 11)^{(x^2-11x+30)} = 1.$$

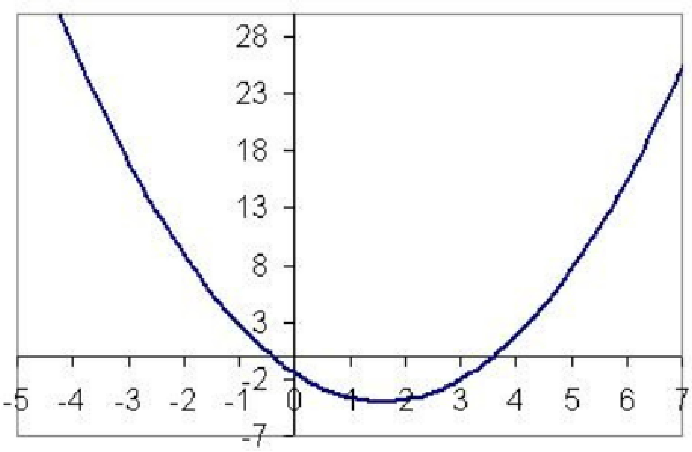
*You may also be interested in the problem Mega Quadratic Equations.*

**suggested solutions: https://nrich.maths.org/1838/solution**

# Root Hunter

**Age 16 to 18**
**Challenge Level** ★

Take a look at the function in the graph below.




The graph is positive for $x = 5$ and negative for $x = 3$. This means that the graph must cut the $x$ axis somewhere between $3$ and $5$.
Although in this case the result is obvious (because we have the whole graph to look at!), we can also use this idea to show that more tricky functions also have roots.

Use this idea to show that these functions possess at least one solution $f(x) = 0$:

$$f(x) = \frac{1}{x-2} + \frac{1}{x-3}$$

$$f(x) = x^x - 1.5x$$

$$f(x) = x^{1000000} + 1000000^x - 17$$

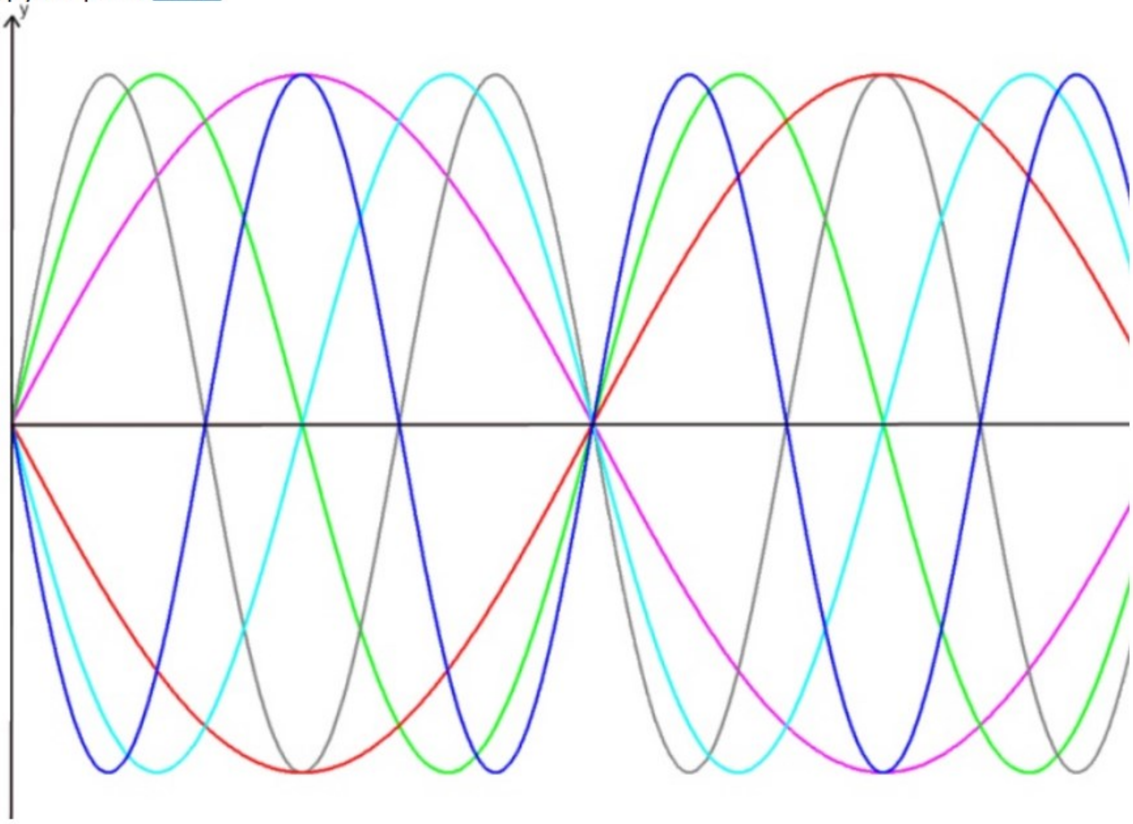
$$f(x) = \cos(\sin(\cos x)) - \sin(\cos(\sin x))$$

**suggested solutions: https://nrich.maths.org/5876/solution**

# Tangled Trig Graphs

**Age 16 to 18**
**Challenge Level** ★

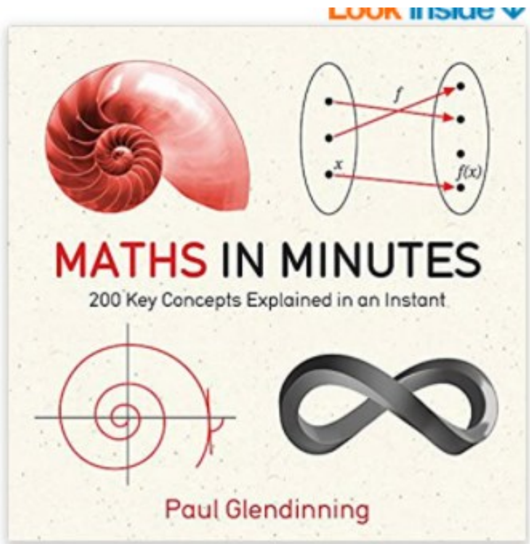
Here is a pattern I made with some graphs of trigonometric functions.You can find a copy to print here.




**suggested solutions**: https://nrich.maths.org/6481/solution

# Just for fun- Additional suggested reading if you are interested in Maths.

## Maths in Minutes: 200 Key Concepts Explained In An Instant Paperback



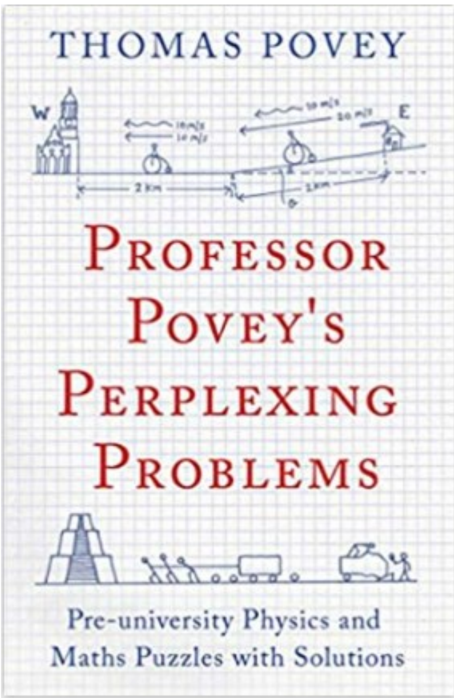

Maths in Minutes is a visually led introduction to 200 key mathematical ideas. Each concept is quick and easy to remember, described by means of an easy-to-understand picture and a maximum 200-word explanation. Concepts span all of the key areas of mathematics, including Fundamentals of Mathematics, Sets and Numbers, Geometry, Equations, Limits, Functions and Calculus, Vectors and Algebra, Complex Numbers, Combinatorics, Number Theory, Metrics and Measures and Topology. Incredibly quick - clear artworks and simple explanations that can be easily remembered. Based on scientific research that the brain best absorbs information visually. Compact and portable format - the ideal, handy reference.

## Professor Povey's Perplexing Problems: Pre-University Physics and Maths Puzzles with Solutions

In *Professor Povey's Perplexing Problems*, Thomas Povey shares 109 of his favourite problems in physics and maths. A tour de force of imagination and exposition, he takes us by the hand and guides us through uncompromisingly challenging territory that expands our minds and encourages a playful and exploratory approach to study. The puzzles, he says, are like toys. We should pick up the one we most enjoy, and play with it.

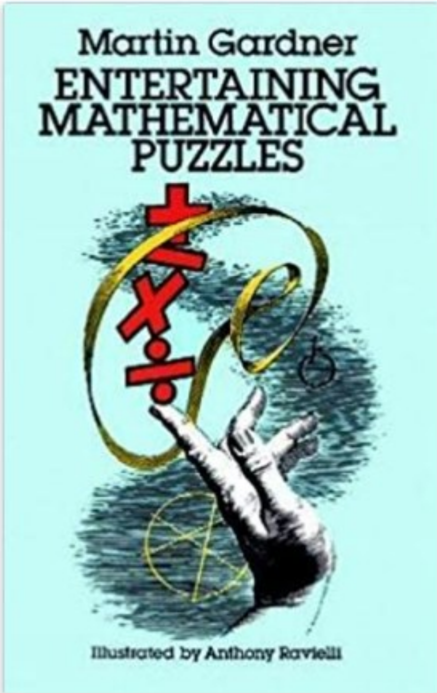
Whether you are an aspiring scientist or an old-hand, pitting yourself against these problems will test your ability to think, and inspire you with curiosity and enthusiasm for physics. Presented with charm and wit, the questions span the gap between high-school and university-entrance standard material. Detailed answers are lightened with a fascinating and refreshing blend of scientific history, application and personal anecdote.

On this delightful and idiosyncratic romp through pre-university maths and physics, the author shows us that behind every single one of these questions lies a new way of thinking about subjects we thought we had understood. He argues that engaging with the unfamiliar is key to forming deeper insights and developing intellectual independence. *Professor Povey's Perplexing Problems* is a manifesto that science should be playful, and a celebration of the curious.

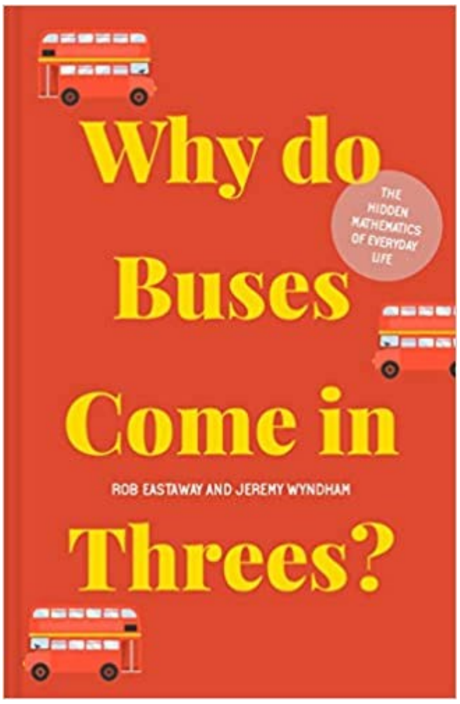


## Entertaining Mathematical Puzzles

Only an elementary knowledge of math is needed to enjoy this entertaining compilation of brain-teasers. It includes a mixture of old and new riddles covering a variety of mathematical topics: money, speed, plane and solid geometry, probability, topology, tricky puzzles and more. Carefully explained solutions follow each problem. 65 black-and-white illustrations.




## Why do Buses Come in Threes?: The hidden mathematics of everyday life




Why is it better to buy a lottery ticket on Friday? Why are showers always too hot or too cold? And which classic puzzle was destroyed by Allied bombing in the war? These and many other questions are answers in this entertaining and highly informative book.

Why do Buses Come in Threes? is for anyone who wants to remind themselves – or discover for the first time – that maths is relevant to almost everything we do. Dating, cooking, travelling by car, gambling and life-saving techniques all have links with intriguing mathematical problems that you will find explained here – including the odd coincidence of 4 July, the exponential growth of Australian rabbits and a surprising formula for running in the rain without getting wet. Whether you have a degree in astrophysics or haven't touched maths since you left school, this book will change the way you view the world around you.

# Answers– Discriminant and Complex numbers

1)

$x^2 - 5x + 7 = 0$

$a = 1, b = -5, c = 7$

So discriminant $= b^2 - 4ac$

$= (-5)^2 - 4(1)(7) = 25 - 28$

$= -3$

$-3 < 0$ so no real solutions.

$7 - 2x - 3x^2 = 0$

$a = -3, b = -2, c = 7$

So discriminant $= b^2 - 4ac$

$= (-2)^2 - 4(7)(-3) = 4 + 84$

$= 88$

$88 > 0$ so two (distinct) real solutions.

$4x^2 - 28x + 49 = 0$

$a = 4, b = -28, c = 49$

So discriminant $= b^2 - 4ac$

$= (-28)^2 - 4(4)(49) = 784 - 784$

$= 0$

So one real solution (coincidental solutions).

2)

$y = 7x^2 - 5x + 4$ since

$a = 7, b = -5, c = 4$

So $b^2 - 4ac = (-5)^2 - 4\times7\times4$

$= -87$

$-87 < 0$ so no real solutions and the curve has a $\cup$ shape

$y = -4x^2 + 12x - 9$ since

$a = -4, b = 12, c = -9$

So $b^2 - 4ac = 12^2 - 4\times(-4)\times(-9)$

$= 0$

so one real solution

$y = 6x^2 - x - 15$ since

$a = 6, b = -1, c = -15$

So $b^2 - 4ac = (-1)^2 - 4\times6\times(-15)$

$= 361$

$361 > 0$ so two real solutions

$y = -x^2 + 2x - 4$ since

$a = -1, b = 2, c = -4$

So $b^2 - 4ac = 2^2 - 4\times(-1)\times(-4)$

$= -12$

$-12 < 0$ so no real solutions and the curve has a $\cap$ shape

3)

**a** $3x^2 + 2x - k = 0$

$a = 3, b = 2, c = -k$

$b^2 - 4ac = 2^2 - 4\times3\times(-k)$

$= 4 + 12k$

Exactly one solution so

$b^2 - 4ac = 0 \Rightarrow 4 + 12k = 0$

$k = -\frac{1}{3}$

**b** $kx^2 - x + 4 = 0$

$a = k, b = -1, c = 4$

$b^2 - 4ac = (-1)^2 - 4\times k\times4$

$= 1 - 16k$

Exactly one solution so

$b^2 - 4ac = 0 \Rightarrow 1 - 16k = 0$

$k = \frac{1}{16}$

3c)

$2x^2 + 5x + k - 5 = 0$

$a = 2, b = 5, c = k - 5$

$b^2 - 4ac = 5^2 - 4\times2\times(k-5)$

$= 65 - 8k$

Exactly one solution so

$b^2 - 4ac = 0 \Rightarrow 65 - 8k = 0$

$k = \frac{65}{8}$

4)

**a** $x^2+3x-3k=0$

$a=1,\ b=3,\ c=-3k$

$b^2-4ac=3^2-4\times1\times(-3k)$

$=9+12k$

Real solutions so

$b^2-4ac\geq0\Rightarrow 9+12k\geq0$

$k\geq-\frac{3}{4}$

**b** $kx^2-7x+4=0$

$a=k,\ b=-7,\ c=4$

$b^2-4ac=(-7)^2-4\times k\times4$

$=49-16k$

Real solutions so

$b^2-4ac\geq0\Rightarrow 49-16k\geq0$

$k\leq\frac{49}{16}$

**c** $-x^2+6x-k-2=0$

$a=-1,\ b=6,\ c=-k-2$

$b^2-4ac=6^2-4\times(-1)\times(-k-2)$

$=28-4k$

Real solutions so

$b^2-4ac\geq0\Rightarrow 28-4k\geq0$

$k\leq7$

5)

**a** $5x^2-x+2k=0$

$a=5,\ b=-1,\ c=2k$

$b^2-4ac=(-1)^2-4\times5\times2k$

$=1-40k$

No real solutions so

$b^2-4ac<0\Rightarrow 1-40k<0$

$k>\frac{1}{40}$

**b** $-kx^2+4x+5=0$

$a=-k,\ b=4,\ c=5$

$b^2-4ac=4^2-4\times(-k)\times5$

$=16+20k$

No real solutions so

$b^2-4ac<0\Rightarrow 16+20k<0$

$k<-\frac{4}{5}$

**c** $6x^2-5x+3-2k=0$

$a=6,\ b=-5,\ c=3-2k$

$b^2-4ac=(-5)^2-4\times6\times(3-2k)$

$=-47+48k$

No real solutions so

$b^2-4ac<0\Rightarrow -47+48k<0$

$k<\frac{47}{48}$

# Answers– Circles

## Section A

**a** $x^2 + y^2 = 25$  **b** $(x - 1)^2 + (y - 3)^2 = 4$  **c** $(x - 4)^2 + (y + 6)^2 = 1$

**d** $(x + 1)^2 + (y + 8)^2 = 9$  **e** $(x + \frac{1}{2})^2 + (y - \frac{1}{2})^2 = \frac{1}{4}$  **f** $(x + 3)^2 + (y - 9)^2 = 12$

**a** centre (0, 0) radius 4  **b** centre (6, 1) radius 9  **c** centre (–1, 4) radius 11

**d** centre (7, 0) radius 0.3  **e** centre (–2, –5) radius $4\sqrt{2}$  **f** centre (8, –9) radius $6\sqrt{3}$

## Section B

**a** $x^2 + (y - 2)^2 - 4 + 3 = 0$
$x^2 + (y - 2)^2 = 1$
centre (0, 2) radius 1

**b** $(x - 1)^2 - 1 + (y - 5)^2 - 25 - 23 = 0$
$(x - 1)^2 + (y - 5)^2 = 49$
centre (1, 5) radius 7

**c** $(x + 6)^2 - 36 + (y - 4)^2 - 16 + 36 = 0$
$(x + 6)^2 + (y - 4)^2 = 16$
centre (–6, 4) radius 4

**d** $(x - 1)^2 - 1 + (y + 8)^2 - 64 = 35$
$(x - 1)^2 + (y + 8)^2 = 100$
centre (1, –8) radius 10

**e** $(x - 4)^2 - 16 + (y + 3)^2 - 9 = 0$
$(x - 4)^2 + (y + 3)^2 = 25$
centre (4, –3) radius 5

**f** $(x + 5)^2 - 25 + (y - 1)^2 - 1 - 19 = 0$
$(x + 5)^2 + (y - 1)^2 = 45$
centre (–5, 1) radius $3\sqrt{5}$

**g** $x^2 + y^2 - x - 6y + \frac{1}{4} = 0$
$(x - \frac{1}{2})^2 - \frac{1}{4} + (y - 3)^2 - 9 + \frac{1}{4} = 0$
$(x - \frac{1}{2})^2 + (y - 3)^2 = 9$
centre $(\frac{1}{2}, 3)$ radius 3

**h** $x^2 + y^2 + \frac{2}{3}x - \frac{8}{3}y + \frac{8}{9} = 0$
$(x + \frac{1}{3})^2 - \frac{1}{9} + (y - \frac{4}{3})^2 - \frac{16}{9} + \frac{8}{9} = 0$
$(x + \frac{1}{3})^2 + (y - \frac{4}{3})^2 = 1$
centre $(-\frac{1}{3}, \frac{4}{3})$ radius 1

## Section C

**a** sub. $x^2 + (x - 4)^2 = 10$
$x^2 - 4x + 3 = 0$
$(x - 1)(x - 3) = 0$
$x = 1, 3$
$\therefore$ (1, –3) and (3, –1)

**b** sub. $y = 17 - 3x$
$x^2 + (17 - 3x)^2 - 4x - 2(17 - 3x) - 15 = 0$
$x^2 - 10x + 24 = 0$
$(x - 4)(x - 6) = 0$
$x = 4, 6$
$\therefore$ (4, 5) and (6, –1)

**c** sub.
$4x^2 + 4(2x + 2)^2 + 4x - 8(2x + 2) - 15 = 0$
$4x^2 + 4x - 3 = 0$
$(2x + 3)(2x - 1) = 0$
$x = -\frac{3}{2}, \frac{1}{2}$
$\therefore$ $(-\frac{3}{2}, -1)$ and $(\frac{1}{2}, 3)$

sub.
$x^2 + (1 - x)^2 + 6x + 2(1 - x) = 27$
$x^2 + x - 12 = 0$
$(x + 4)(x - 3) = 0$
$x = -4, 3$
$\therefore$ (–4, 5) and (3, –2)
$AB = \sqrt{49 + 49} = 7\sqrt{2}$

# Task 3 : Factor theorem and Polynomial division—ANSWERS

## Section A

**a** let $f(x) \equiv x^3 + 2x^2 - 2x - 1$
$f(1) = 1 + 2 - 2 - 1 = 0$
$\therefore (x - 1)$ is a factor

**b** let $f(x) \equiv x^3 - 5x^2 - 9x + 2$
$f(-2) = -8 - 20 + 18 + 2 = -8$
$\therefore (x + 2)$ is not a factor

**c** let $f(x) \equiv x^3 - x^2 - 14x + 27$
$f(3) = 27 - 9 - 42 + 27 = 3$
$\therefore (x - 3)$ is not a factor

**d** let $f(x) \equiv 2x^3 + 13x^2 + 2x - 24$
$f(-6) = -432 + 468 - 12 - 24 = 0$
$\therefore (x + 6)$ is a factor

**e** let $f(x) \equiv 2x^3 - 5x^2 + 7x - 14$
$f(-\frac{1}{2}) = -\frac{1}{4} - \frac{5}{4} - \frac{7}{2} - 14 = -19$
$\therefore (2x + 1)$ is not a factor

**f** let $f(x) \equiv 2 - 17x + 25x^2 - 6x^3$
$f(\frac{2}{3}) = 2 - \frac{34}{3} + \frac{100}{9} - \frac{16}{9} = 0$
$\therefore (3x - 2)$ is a factor

**a** $f(1) = 1 - 2 - 11 + 12 = 0$
$\therefore (x - 1)$ is a factor of $f(x)$

**b**

$$\begin{array}{r} x^2 - x - 12 \\ x - 1 \enclose{longdiv}{x^3 - 2x^2 - 11x + 12} \\ \underline{x^3 - x^2 \phantom{- 11x + 12}} \\ -x^2 - 11x \phantom{+ 12} \\ \underline{-x^2 + x \phantom{+ 12}} \\ -12x + 12 \\ \underline{-12x + 12} \end{array}$$

$\therefore f(x) \equiv (x - 1)(x^2 - x - 12)$
$\equiv (x - 1)(x + 3)(x - 4)$

$g(-3) = -54 + 9 + 39 + 6 = 0$
$\therefore (x + 3)$ is a factor of $g(x)$

$$\begin{array}{r} 2x^2 - 5x + 2 \\ x + 3 \enclose{longdiv}{2x^3 + x^2 - 13x + 6} \\ \underline{2x^3 + 6x^2 \phantom{- 13x + 6}} \\ -5x^2 - 13x \phantom{+ 6} \\ \underline{-5x^2 - 15x \phantom{+ 6}} \\ 2x + 6 \\ \underline{2x + 6} \end{array}$$

$\therefore g(x) \equiv (x + 3)(2x^2 - 5x + 2)$
$\equiv (x + 3)(2x - 1)(x - 2)$

$g(x) = 0 \Rightarrow (x + 3)(2x - 1)(x - 2) = 0$
$x = -3, \frac{1}{2}$ or $2$

## Section B

**a**

$$\begin{array}{r} x^2 + x - 2 \\ x + 1 \enclose{longdiv}{x^3 + 2x^2 - x - 2} \\ \underline{x^3 + x^2 \phantom{- x - 2}} \\ x^2 - x \phantom{- 2} \\ \underline{x^2 + x \phantom{- 2}} \\ -2x - 2 \\ \underline{-2x - 2} \end{array}$$

quotient: $x^2 + x - 2$

**b**

$$\begin{array}{r} x^2 + 4x - 1 \\ x - 2 \enclose{longdiv}{x^3 + 2x^2 - 9x + 2} \\ \underline{x^3 - 2x^2 \phantom{- 9x + 2}} \\ 4x^2 - 9x \phantom{+ 2} \\ \underline{4x^2 - 8x \phantom{+ 2}} \\ -x + 2 \\ \underline{-x + 2} \end{array}$$

quotient: $x^2 + 4x - 1$

**c**

$$
\begin{array}{r|l}
 & x^2 - x + 5 \\
\hline
x+4 & x^3 + 3x^2 + x + 20 \\
 & \underline{x^3 + 4x^2} \\
 & \quad -x^2 + x \\
 & \quad \underline{-x^2 - 4x} \\
 & \qquad\quad 5x + 20 \\
 & \qquad\quad \underline{5x + 20}
\end{array}
$$

quotient: $x^2 - x + 5$

**d**

$$
\begin{array}{r|l}
 & 2x^2 + x - 3 \\
\hline
x-1 & 2x^3 - x^2 - 4x + 3 \\
 & \underline{2x^3 - 2x^2} \\
 & \qquad x^2 - 4x \\
 & \qquad \underline{x^2 - x} \\
 & \qquad\quad -3x + 3 \\
 & \qquad\quad \underline{-3x + 3}
\end{array}
$$

quotient: $2x^2 + x - 3$

**e**

$$
\begin{array}{r|l}
 & 6x^2 + 11x - 18 \\
\hline
x-5 & 6x^3 - 19x^2 - 73x + 90 \\
 & \underline{6x^3 - 30x^2} \\
 & \qquad 11x^2 - 73x \\
 & \qquad \underline{11x^2 - 55x} \\
 & \qquad\quad -18x + 90 \\
 & \qquad\quad \underline{-18x + 90}
\end{array}
$$

quotient: $6x^2 + 11x - 18$

**f**

$$
\begin{array}{r|l}
 & -x^2 + 7x - 4 \\
\hline
x+2 & -x^3 + 5x^2 + 10x - 8 \\
 & \underline{-x^3 - 2x^2} \\
 & \qquad 7x^2 + 10x \\
 & \qquad \underline{7x^2 + 14x} \\
 & \qquad\quad -4x - 8 \\
 & \qquad\quad \underline{-4x - 8}
\end{array}
$$

quotient: $-x^2 + 7x - 4$

**g**

$$
\begin{array}{r|l}
 & x^2 - 3x + 7 \\
\hline
x+3 & x^3 + 0x^2 - 2x + 21 \\
 & \underline{x^3 + 3x^2} \\
 & \quad -3x^2 - 2x \\
 & \quad \underline{-3x^2 - 9x} \\
 & \qquad\quad 7x + 21 \\
 & \qquad\quad \underline{7x + 21}
\end{array}
$$

quotient: $x^2 - 3x + 7$

**h**

$$
\begin{array}{r|l}
 & 3x^2 - 2x + 12 \\
\hline
x+6 & 3x^3 + 16x^2 + 0x + 72 \\
 & \underline{3x^3 + 18x^2} \\
 & \quad -2x^2 + 0x \\
 & \quad \underline{-2x^2 - 12x} \\
 & \qquad\quad 12x + 72 \\
 & \qquad\quad \underline{12x + 72}
\end{array}
$$

quotient: $3x^2 - 2x + 12$

## Task 4 : Radians ANSWERS

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | a $\pi$ | b $\frac{\pi}{6}$ | c $\frac{\pi}{4}$ | d $4\pi$ | e $\frac{\pi}{10}$ | f $\frac{2\pi}{3}$ |
| | g $\frac{\pi}{12}$ | h $\frac{2\pi}{9}$ | i $\frac{3\pi}{2}$ | j $\frac{\pi}{24}$ | k $\frac{4\pi}{5}$ | l $\frac{11\pi}{9}$ |
| 2 | a $0.17^c$ | b $0.66^c$ | c $5.08^c$ | d $1.11^c$ | e $8.85^c$ | f $2.20^c$ |
| 3 | a 360° | b 60° | c 90° | d 135° | e 10° | f 6° |
| | g 150° | h 22.5° | i 540° | j 24° | k 420° | l 81° |
| 4 | a 114.6° | b 28.6° | c 177.6° | d 81.9° | e 498.5° | f 42.5° |

$$\tan(225°) = \mathbf{\tan(45°) = 1}$$

$$\tan(210°) = \mathbf{\tan(30°) = \frac{1}{\sqrt{3}}}$$

$$\sin(150°) = \mathbf{\sin(30°) = \frac{1}{2}}$$

$$\cos(300°) = \mathbf{\cos(60°) = \frac{1}{2}}$$

$$\sin(-45°) = -\mathbf{\sin(45°) = -\frac{1}{\sqrt{2}}}$$

$$\cos(750°) = \mathbf{\cos(30°) = \frac{\sqrt{3}}{2}}$$

$$\cos(120°) = -\mathbf{\cos(60°) = -\frac{1}{2}}$$