Maths Induction Pack 2025

$u_n = ar^{n-1}$

$\log_a x = \frac{\log_b x}{\log_b a}$

Welcome!

First of all, welcome to Maths! This pack is designed to support your recall of GCSE topics essential for Alevel maths.

A level Mathematics is an interesting and challenging course which extends the methods you learned at GCSE and includes applications of mathematics, such as Statistics and Mechanics. Mathematics qualifications are well-respected by employers and are "facilitating" subjects for entry to higher education. As technology continues to advance, so too does mathematics, increasing its essential role in both everyday and corporate life. New mathematical theories are being discovered and developed every day, enabling inventions and scientific discoveries to continue to flourish. By studying Maths at A-Level, you will have the opportunity to forge a career that's at the forefront of technological advancement.

A Level Maths will allow you to develop a number of skills : problem-solving, analytical skills, research skills and logic whilst letting you hone your ability to solve mathematical problems that are both abstract and scientific.

This induction pack includes tasks which form part of the assessment to judge your suitability to access the course in September. To access the course you need to complete these induction tasks thoroughly and pass the baseline assessments in September.

If you get stuck on any of the tasks, feel free to email me or any of the maths team .
michelle.cole@bushfield.co.uk

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

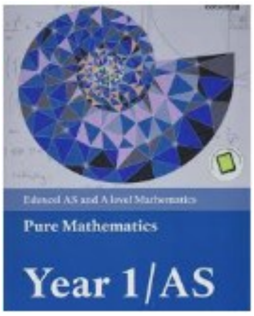
$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B \mid A)P(A) + P(B \mid A')P(A')}$$

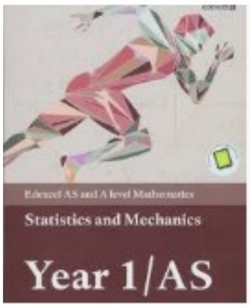
$$u_n = a + (n-1)d$$

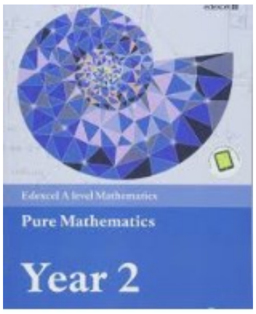
Key Information for A Level Maths

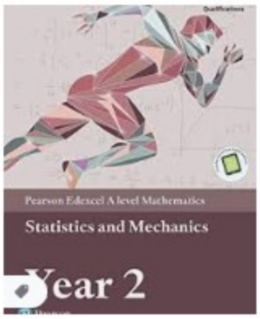
Specification: Pearson https://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2017/specification-and-sample-assesment/a-level-l3-mathematics-specification.pdf

Textbooks– will be provided by the school









Assessment: 3 x 2hour papers.

Two papers on the Pure maths content and one paper on the Applied maths content split equally between statistics and mechanics.

Content Over-view

PURE :

- Topic 1 – Proof
- Topic 2 – Algebra and functions
- Topic 3 – Coordinate geometry in the (*x, y*) plane
- Topic 4 – Sequences and series
- Topic 5 – Trigonometry
- Topic 6 – Exponentials and logarithms
- Topic 7 – Differentiation
- Topic 8 – Integration
- Topic 9 – Numerical methods
- Topic 10 – Vectors

Applied:

Statistics

- Topic 1 – Statistical sampling
- Topic 2 – Data presentation and interpretation
- Topic 3 – Probability
- Topic 4 – Statistical distributions
- Topic 5 – Statistical hypothesis testing

Mechanics

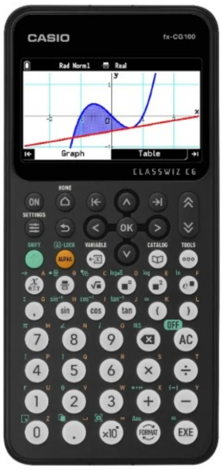
- Topic 6 – Quantities and units in mechanics
- Topic 7 – Kinematics

Organisational Requirements

You are required to have your own calculator that has certain functions available for use during all lessons, independent study and your exams. The most popular one for A Level maths is shown.

Casio FX -991 CW Advanced Scientific calculator

Casio fx- CG100(class whiz) graphical calculator



Early in the autumn term we will arrange purchase via wise pay for your chosen calculator as we can buy these cheaper

You may choose to work on loose leaf paper and store your work in Lever Arch folders or A4 books for working will be provided.

A Level Maths Preparatory Tasks

You should work through each of the tasks shared in this booklet, ensuring that you are confident with all of the skills.

Please ensure that you keep all of your work demonstrating full working out to bring to the first session in September.

The solutions to all tasks are shared at the end of this booklet. You should self check your work.

The ' careful reading ' Introductory task

Follow the instructions on page 23 to complete the reading task. If you found this article interesting then you can find plenty more at: https://plus.maths.org/content/

Mathematical Problem Solving

A selection of mathematical puzzles aimed at transition into A Level for you to have a go at. Links to solutions shared.

Just for fun- Additional suggested reading if you are interested in Maths

Some suggested reading if you want to further explore the beauty of maths.

Task A: Indices and Surds

Example 1

Simplify these expressions. **a** $2x^3 \times 3x^5$ **b** $12x^7 \div 4x^3$ **c** $(3x^5)^3$

a $2x^3 \times 3x^5 = 6x^{3+5}$

$= 6x^8$

Multiply the coefficients together and use $x^a \times x^b = x^{a+b}$

b $12x^7 \div 4x^6 = \dfrac{12x^7}{4x^6}$

$= 3x$

Since $\dfrac{12}{4} = 3$ and $x^a \div x^b = x^{a-b}$ so $\dfrac{x^7}{x^6} = x^1$ which we just write as x

c $(3x^5)^3 = 3^3(x^5)^3$

Both the 3 and the x^5 must be raised to the power 3

$= 27x^{15}$

Since $(x^a)^b = x^{ab}$

Example 2

Evaluate each of these without using a calculator.

a $25^{0.5}$ **b** 6^{-2} **c** $8^{\frac{2}{3}}$

a $25^{0.5} = 25^{\frac{1}{2}}$

$= \sqrt{25}$

Since a power of $\frac{1}{2}$ represents a square root.

$= 5$

b $6^{-2} = (6^2)^{-1}$

$= \dfrac{1}{6^2}$

Since a power of −1 represents a reciprocal.

$= \dfrac{1}{36}$

c $8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2$

Always calculate a root before a power.

$= 2^2$

Since the cube root of 8 is 2

$= 4$

Example 3

Write these expressions in simplified index form.

a $\sqrt[3]{x}$ **b** $\dfrac{2}{x^3}$ **c** $\dfrac{2x}{\sqrt{x}}$

a $\sqrt[3]{x} = x^{\frac{1}{3}}$

b $\dfrac{2}{x^3} = 2x^{-3}$

c $\dfrac{2x}{\sqrt{x}} = \dfrac{2x}{x^{\frac{1}{2}}}$

Since $\sqrt{x} = x^{\frac{1}{2}}$

$= 2x^{1-\frac{1}{2}}$

Subtract the powers, remembering that $x = x^1$

$= 2x^{\frac{1}{2}}$

Example 4

Simplify these expressions without using a calculator.

a $\sqrt{18}+5\sqrt{2}$ **b** $\frac{6}{\sqrt{3}}$ **c** $\frac{2}{1-\sqrt{5}}$

a $\sqrt{18}=\sqrt{9}\sqrt{2}$

$=3\sqrt{2}$

9 is a square-number factor of 18 so you can simplify $\sqrt{18}$

Therefore $\sqrt{18}+5\sqrt{2}=3\sqrt{2}+5\sqrt{2}$

$=8\sqrt{2}$

Collect like terms.

b $\frac{6}{\sqrt{3}}=\frac{6\sqrt{3}}{\sqrt{3}\sqrt{3}}$

Rationalise the denominator by multiplying numerator and denominator by $\sqrt{3}$

$=\frac{6\sqrt{3}}{3}$

$=2\sqrt{3}$

Since $6 \div 3 = 2$

c $\frac{2}{1-\sqrt{5}}=\frac{2(1+\sqrt{5})}{(1-\sqrt{5})(1+\sqrt{5})}$

Rationalise the denominator by multiplying numerator and denominator by $1+\sqrt{5}$

$=\frac{2(1+\sqrt{5})}{-4}$

$(1-\sqrt{5})(1+\sqrt{5})=1-\sqrt{5}+\sqrt{5}-5$

$=1-5=-4$

$=-\frac{1}{2}(1+\sqrt{5})$

1 Evaluate each of these without using a calculator.

a $49^{\frac{1}{2}}$

b $27^{\frac{1}{3}}$

c 5^{-1}

d $64^{-\frac{1}{3}}$

e $9^{\frac{3}{2}}$

f $16^{\frac{3}{4}}$

g $125^{-\frac{2}{3}}$

h $\left(\frac{1}{2}\right)^{3}$

i $\left(\frac{1}{9}\right)^{-2}$

j $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

k $\left(\frac{9}{16}\right)^{-0.5}$

l $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

2 Simplify these expressions fully without using a calculator.

a $\sqrt{8}$

b $\sqrt{75}$

c $2\sqrt{24}$

d $3\sqrt{48}$

e $\sqrt{20}+\sqrt{5}$

f $\sqrt{27}-\sqrt{12}$

g $5\sqrt{32}-3\sqrt{8}$

h $\sqrt{50}+3\sqrt{125}$

i $\sqrt{68}+3\sqrt{17}$

j $3\sqrt{72}-\sqrt{32}$

k $4\sqrt{18}-2\sqrt{3}$

l $6\sqrt{5}+\sqrt{50}$

3 Simplify these expressions fully without using a calculator.

a $\frac{1}{\sqrt{7}}$

b $\frac{2}{\sqrt{8}}$

c $\frac{12}{\sqrt{3}}$

d $\frac{\sqrt{8}}{\sqrt{12}}$

e $\frac{1}{1+\sqrt{3}}$

f $\frac{2}{1+\sqrt{2}}$

g $\frac{8}{1-\sqrt{5}}$

h $\frac{2}{\sqrt{5}-1}$

i $\frac{\sqrt{2}}{2+\sqrt{3}}$

j $\frac{2\sqrt{3}}{\sqrt{6}-2}$

k $\frac{1+\sqrt{2}}{1-\sqrt{2}}$

l $\frac{3+\sqrt{5}}{\sqrt{5}-3}$

4 Expand the brackets and fully simplify each expression.

a $(1+\sqrt{2})(3+\sqrt{2})$

b $(1+\sqrt{2})(3-\sqrt{2})$

c $(1-\sqrt{2})(3+\sqrt{2})$

d $(1-\sqrt{2})(3-\sqrt{2})$

e $(\sqrt{3}+2)(4+\sqrt{3})$

f $(\sqrt{3}+2)(4-\sqrt{3})$

g $(\sqrt{3}-2)(4+\sqrt{3})$

h $(\sqrt{3}-2)(4-\sqrt{3})$

i $(\sqrt{6}+1)(\sqrt{2}+3)$

j $(\sqrt{6}+1)(\sqrt{2}-3)$

k $(\sqrt{6}-1)(\sqrt{2}+3)$

l $(\sqrt{6}-1)(\sqrt{2}-3)$

5 Write each of these expressions in simplified index form.

a $x^3 \times x^7$

b $7x^5 \times 3x^6$

c $5x^4 \times 8x^7$

d $x^8 \div x^2$

e $8x^7 \div 2x^9$

f $3x^8 \div 12x^7$

g $(x^5)^7$

h $(x^2)^{-5}$

i $(3x^2)^4$

j $(6x^5)^2$

k $\sqrt{x^3}$

l $\sqrt[4]{x^5}$

m $\dfrac{5\sqrt{x}}{x}$

n $2x\sqrt{x}$

o $\dfrac{x^2}{3\sqrt{x}}$

p $x^3(x^5 - 1)$

q $x^3(\sqrt{x} + 2)$

r $\dfrac{x+2}{x^3}$

s $\dfrac{\sqrt{x}+3}{x}$

t $\dfrac{(3-x^3)}{\sqrt{x}}$

u $(\sqrt{x}+3)^2$

v $\dfrac{3+\sqrt{x}}{x^2}$

w $\dfrac{1-x}{2\sqrt{x}}$

x $\dfrac{\sqrt{x}+2}{3x^3}$

Task B: Equations and Formula

Example 1

Solve the equation $7x-5=3x-2$

Step	Explanation
$4x-5=-2$	Subtract $3x$ from both sides of the equation.
$4x=3$	Add 5 to both sides of the equation.
$x=\frac{3}{4}$	Divide both sides of the equation by 4

Example 2

Solve the inequality $5(x-2)\leq 2x+1$

Step	Explanation
$5x-10\leq 2x+1$	First expand the brackets.
$3x-10\leq 1$	Subtract $2x$ from both sides.
$3x\leq 11$	Add 10 to both sides.
$x\leq\frac{11}{3}$	Divide both sides by 3

Example 3

Rearrange $Ax-3=\frac{x+B}{2}$ to make x the subject.

Step	Explanation
$2Ax-6=x+B$	Multiply both sides by 2
$2Ax-6-x=B$	Subtract x from both sides.
$2Ax-x=B+6$	Add 6 to both sides.
$x(2A-1)=B+6$	Factorise the side involving x
$x=\frac{B+6}{2A-1}$	Divide both sides by $(2A-1)$ to make x the subject.

Example 4

Solve the simultaneous equations $5x-4y=17$, $3x+8y=5$

Step	Explanation
$15x+40y=25$ (1)	Multiply the second equation by 5
$15x-12y=51$ (2)	Multiply the first equation by 3
(1)−(2): $52y=-26$	Subtract equation (2) from equation (1) to eliminate x
$y=-\frac{1}{2}$	
$5x-4\left(-\frac{1}{2}\right)=17$	Substitute $y=-\frac{1}{2}$ into one of the original equations.
$5x+2=17$	Solve this equation to find the value of x
$5x=15$	
$x=3$	

Example 5

Find the point of intersection between the lines with equations $y=2x+5$ and $y=7-3x$

Step	Explanation
$2x+5=7-3x$	Substitute $2x+5$ for y in the second equation.
$5x+5=7$	Solve to find the value of x
$5x=2$	
$x=0.4$	
$y=2(0.4)+5$	Substitute $x=0.4$ into either of the original equations to find the y-coordinate.
$=5.8$	

So the lines intersect at the point (0.4, 5.8)

1 Solve each of these linear equations.

a $3(2x+9)=7$

b $7-3x=12$

c $\frac{x+4}{5}=7$

d $2x+7=5x-6$

e $8x-3=2(3x+1)$

f $\frac{2x+9}{12}=x-1$

g $2(3x-7)=4x$

h $7-2x=3(4-5x)$

2 Solve each of these linear inequalities.

a $\frac{x}{2}+7\geq 5$

b $3-4x<15$

c $5(x-1)>12+x$

d $\frac{x+1}{3}>2$

e $8x-1\leq 2x-5$

f $3(x+1)\geq\frac{x-3}{2}$

g $3(2x-5)<1-x$

h $x-(3+2x)\geq 2(x+1)$

3 Rearrange each of these formulae to make x the subject.

a $2x+5=3A-1$

b $x+u=vx+3$

c $\frac{3x-1}{k}=2x$

d $5(x-3m)=2nx-4$

e $(1-3x)^2=t$

f $\frac{1}{x}=\frac{1}{p}+\frac{1}{q}$

g $\frac{1}{x^2+k}-6=4$

h $\sqrt{x+A}=2B$

4 Use algebra to solve each of these pairs of simultaneous equations.

a $5x+12y=-6,\ x+5y=4$

b $7x+5y=14,\ 3x+4y=19$

c $2x-5y=4,\ 3x-8y=5$

d $3x-2y=2,\ 8x+3y=4.5$

e $5x-2y=11,\ -2x+3y=22$

f $8x+5y=-0.5,\ -6x+4y=-3.5$

5 Use algebra to find the point of intersection between each pair of lines.

a $y=8-3x,\ y=2-5x$

b $y=7x-4,\ y=3x-2$

c $y=2x+3,\ y=5-x$

d $y+5=3x,\ y=-5x+7$

e $y=\frac{1}{2}x+3,\ y=5-2x$

f $y=3(x+2),\ y=7-2x$

Task C: Quadratics

Example 1

Factorise each of these quadratics.

a $9x^2+15x$ **b** $x^2+3x-10$ **c** x^2-16

a $9x^2+15x=3x(3x+5)$ — The highest common factor of $9x^2$ and $15x$ is $3x$

b $x^2+3x-10=(x+5)(x-2)$ — You need to find two constants with a product of -10 and a sum of 3: $5\times-2=-10$ and $5+-2=3$ so the constants are -2 and 5

c $x^2-16=(x+4)(x-4)$ — x^2 and 16 are both square numbers.

Example 2

Factorise each of these quadratics.

a $3x^2+11x+6$ **b** $2x^2-9x+10$

a $3x^2+11x+6=3x^2+9x+2x+6$ — Split $11x$ into $9x+2x$ since $9\times2=18$ and $3\times6=18$

$=3x(x+3)+2(x+3)$ — Factorise the first pair of terms and the second pair of terms.

$=(3x+2)(x+3)$

b $2x^2-9x+10=2x^2-4x-5x+10$ — Split $9x$ into $-4x-5x$ since $-4\times-5=20$ and $2\times10=20$

$=2x(x-2)-5(x-2)$ — Factorise the first pair of terms and the second pair of terms.

$=(2x-5)(x-2)$

Example 3

Use factorisation to find the roots of these quadratic equations.

a $4x^2+12x=0$ **b** $5x^2=21x-4$

a $4x^2+12x=4x(x+3)$ — Factorise the quadratic.

$4x(x+3)=0\Rightarrow4x=0$ or $x+3=0$ — One of the factors must be equal to zero.

If $4x=0$ then $x=0$ and if $x+3=0$ then $x=-3$ — Solve to find the roots.

b $5x^2-21x+4=0$ — Rearrange so you have a quadratic expression equal to zero.

$5x^2-21x+4=5x^2-20x-x+4$ — Write $-21x=-x-20x$ since $-20\times-1=20$ and $5\times4=20$

$=5x(x-4)-(x-4)$

$=(5x-1)(x-4)$ — Factorise the quadratic.

$(5x-1)(x-4)=0\Rightarrow5x-1=0$ or $x-4=0$ — The product is zero so one of the factors must be equal to zero.

If $5x-1=0$ then $x=\frac{1}{5}$ and if $x-4=0$ then $x=4$ — Solve to find the roots.

1 Fully factorise each of these quadratics.

a $3x^2+5x$

b $8x^2-4x$

c $17x^2+34x$

d $18x^2-24x$

2 Factorise each of these quadratics.

a x^2+5x+6

b $x^2-7x+10$

c x^2-5x-6

d $x^2+3x-28$

e x^2-x-72

f $x^2+2x-48$

g $x^2-12x+11$

h $x^2-5x-24$

3 Factorise each of these quadratics.

a x^2-100

b x^2-81

c $4x^2-9$

d $64-9x^2$

4 Factorise each of these quadratics.

a $3x^2+7x+2$

b $6x^2+17x+12$

c $4x^2-13x+3$

d $2x^2-7x-15$

e $2x^2+3x-5$

f $7x^2+25x-12$

g $8x^2-22x+15$

h $12x^2+17x-5$

5 Fully factorise each of these quadratics.

a $16x^2-25$

b $4x^2-16x$

c $x^2+13x+12$

d $3x^2+16x-35$

e x^2+x-12

f $100-9x^2$

g $2x^2-14x$

h $20x^2-3x-2$

6 Use factorisation to find the roots of these quadratic equations.

a $21x^2-7x=0$

b $x^2-36=0$

c $17x^2+34x=0$

d $6x^2+13x+5=0$

e $4x^2-49=0$

f $x^2=7x+18$

g $x^2-7x+6=0$

h $21x^2=2-x$

i $17x=5x^2+6$

j $16x^2+24x+9=0$

k $9x^2+4=12x$

l $40x^2+x=6$

Example 4

Sketch these quadratic functions.

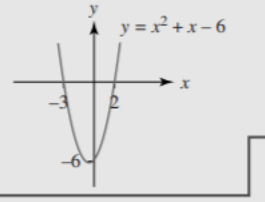
a $y = x^2 + x - 6$ **b** $y = -x^2 + 4x$

a When $x = 0$, $y = -6$ — Find the y-intercept by letting $x = 0$

When $y = 0$, $x^2 + x - 6 = 0$ — Find the x-intercept by letting $y = 0$

$x^2 + x - 6 = (x + 3)(x - 2)$ — Factorise to find the roots.

$(x + 3)(x - 2) = 0 \Rightarrow x = -3$ or $x = 2$ — Sketch the parabola and label the y-intercept of –6 and the x-intercepts of –3 and 2

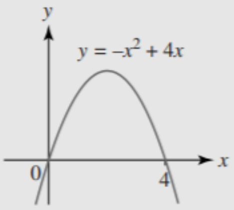


b When $x = 0$, $y = 0$ — Find the y-intercept by letting $x = 0$

When $y = 0$, $-x^2 + 4x = 0$ — Find the x-intercept by letting $y = 0$

$-x^2 + 4x = -x(x - 4)$

$-x(x - 4) = 0 \Rightarrow x = 0$ or $x = 4$ — Factorise to find the roots.



Sketch the parabola, it will be this way up since the x^2 term in the quadratic is negative. Label the x and y intercepts.

7 Sketch each of these quadratic functions, labelling where they cross the x and y axes.

a $y = x(x - 3)$ **b** $y = -x(3x + 2)$ **c** $y = x(3 - x)$

d $y = (x + 2)(x - 2)$ **e** $y = (x + 4)^2$ **f** $y = -(2x + 5)^2$

g $y = (x - 5)(x + 2)$ **h** $y = (x + 1)(5 - x)$

8 Sketch each of these quadratic functions, labelling where they cross the x and y axes.

a $y = x^2 + 6x$ **b** $y = 3x^2 - 12x$ **c** $y = x^2 - 121$

d $y = x^2 - 3x - 10$ **e** $y = -x^2 + 3x$ **f** $y = 15x - 10x^2$

g $y = 49 - x^2$ **h** $y = -x^2 + 2x + 3$ **i** $y = x^2 - 4x + 4$

j $y = -x^2 + 14x - 49$ **k** $y = 3x^2 + 4x + 1$ **l** $y = -2x^2 + 11x - 12$

Task D: Completing the Square

Example 1

Write each of these quadratics in the form $p(x+q)^2+r$ where p, q and r are constants to be found.

a x^2+6x+7 **b** $-2x^2+12x$

a $x^2+6x+7=\left(x+\frac{6}{2}\right)^2-\left(\frac{6}{2}\right)^2+7$

$=(x+3)^2-9+7$

$=(x+3)^2-2$

The constant term in the bracket will be half of the coefficient of x

b $-2x^2+12x=-2[x^2-6x]$

$=-2[(x-3)^2-9]$

$=-2(x-3)^2+18$

First factor out the coefficient of x^2 then complete the square for the expression in the square brackets.

Example 2

Find the coordinates of the turning point of the curve with equation $y=-x^2+5x-2$

$-x^2+5x-2=-[x^2-5x+2]$

First factor out the -1 then complete the square for the expression in the square brackets.

$=-\left[\left(x-\frac{5}{2}\right)^2-\frac{25}{4}+2\right]$

$=-\left[\left(x-\frac{5}{2}\right)^2-\frac{17}{4}\right]$

$=-\left(x-\frac{5}{2}\right)^2+\frac{17}{4}$

So the maximum point is at $\left(\frac{5}{2},\frac{17}{4}\right)$

The curve is at its highest point when the bracket is equal to zero: $x-\frac{5}{2}=0\Rightarrow x=\frac{5}{2}$

1 Write each of these quadratic expressions in the form $p(x+q)^2+r$

a x^2+8x

b x^2-18x

c x^2+6x+3

d $x^2+12x-5$

e $x^2-7x+10$

f x^2+5x+9

g $2x^2+8x+4$

h $3x^2+18x-6$

i $2x^2-10x+3$

j $-x^2+12x-1$

k $-x^2+9x-3$

l $-2x^2+5x-1$

2 Use completing the square to find the turning point of each of these curves and state whether it is a maximum or a minimum.

a $y = x^2 + 14x$

b $y = x^2 - 18x + 3$

c $y = x^2 - 9x$

d $y = -x^2 + 4x$

e $y = x^2 + 11x + 30$

f $y = -x^2 + 6x - 7$

g $y = 2x^2 + 16x - 5$

h $y = -3x^2 + 15x - 2$

Task E: Quadratic Formula

Example 1

Solve the equation $3x^2 - 5x - 7 = 0$ using the quadratic formula.

$a = 3, b = -5, c = -7$ — Substitute into the formula, taking care with negatives.

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 3 \times (-7)}}{2 \times 3}$$

$$= \frac{5 \pm \sqrt{109}}{6}$$ — Use your calculator to give answer as a decimal: $\frac{5 + \sqrt{109}}{6} = 2.57$ and $\frac{5 - \sqrt{109}}{6} = -0.91$

$= 2.57$ or -0.91 (to 2 dp)

You can also use the equation solver on your calculator to solve quadratic equations.

1 Use the quadratic formula to solve each of these equations.

a $7x^2 + 3x - 8 = 0$

b $-x^2 + 4x - 2 = 0$

c $x^2 - 12x + 4 = 0$

Task F: Line Graphs

Example 1

Calculate the gradient of the line through the points $A(1, -6)$ and $B(-5, 2)$

$m = \frac{2-(-6)}{(-5)-1}$

$= \frac{8}{-6}$

$= -\frac{4}{3}$

Use $m = \frac{y_2 - y_1}{x_2 - x_1}$ with $x_1 = 1$, $x_2 = -5$ and $y_1 = -6$, $y_2 = 2$

The line has a negative gradient so slopes down from left to right.

Example 2

Calculate the exact distance between the point $(5, 1)$ and $(6, -4)$

$d = \sqrt{(6-5)^2 + (-4-1)^2}$

$= \sqrt{1^2 + (-5)^2}$

$= \sqrt{26}$

Use $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ with $x_1 = 5$, $x_2 = 6$ and $y_1 = 1$, $y_2 = -4$

Leave answer as a surd since this is **exact**.

Example 3

The points A and B have coordinates $(-4, -9)$ and $(6, -2)$ respectively. Find the midpoint of AB

$\text{Midpoint} = \left(\frac{(-4)+6}{2}, \frac{(-9)+(-2)}{2}\right)$

$= \left(\frac{2}{2}, \frac{-11}{2}\right)$

$= (1, -5.5)$

Use $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ with $x_1 = -4$, $x_2 = 6$ and $y_1 = -9$, $y_2 = -2$

Example 4

Work out the gradient and the y-intercept of each of these lines.

a $y = \frac{1}{2}x + 4$ **b** $y + x = 5$ **c** $-2x + 3y + 7 = 0$

a Gradient $= \frac{1}{2}$ and y-intercept $= 4$

Since $y = mx + c$ where m is the gradient and c is the y-intercept.

b $y = 5 - x$

Rearrange to make y the subject.

So gradient $= -1$ and y-intercept $= 5$

c $3y = -7 + 2x$

Rearrange to make y the subject.

$y = -\frac{7}{3} + \frac{2}{3}x$

So gradient $= \frac{2}{3}$ and y-intercept $= -\frac{7}{3}$

Example 5

Find the equation of the line through the points $(3, 7)$ and $(4, -2)$ in the form $y = mx + c$

$m = \frac{(-2)-7}{4-3}$

$= -9$

First use $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the gradient.

So the equation is $y - 7 = -9(x - 3)$

Use $y - y_1 = m(x - x_1)$ with $(x_1, y_1) = (3, 7)$, or you could use the point $(4, -2)$ instead.

$y - 7 = -9x + 27$

$y = -9x + 34$

Expand the brackets and rearrange to the correct form.

Example 6

The line l_1 has equation $2x+6y=5$. The line l_2 is parallel to l_1 and passes through the point (1, –5). Find the equation of l_2 in the form $ax+by+c=0$ where a, b and c are integers.

$l_1: 2x+6y=5 \Rightarrow 6y=5-2x$

$\Rightarrow y=\frac{5}{6}-\frac{2}{6}x$ — Rearrange to make y the subject so you can see what the gradient is.

The gradient of l_1 is $-\frac{2}{6}$ which simplifies to $-\frac{1}{3}$

Therefore the gradient of l_2 is $-\frac{1}{3}$ — Since l_1 and l_2 are parallel.

So the equation of l_2 is $y-(-5)=-\frac{1}{3}(x-1)$ — Use $y-y_1=m(x-x_1)$ to write the equation of l_2

$\Rightarrow y+5=-\frac{1}{3}(x-1)$

$\Rightarrow -3y-15=x-1$ — Multiply both sides by –3 so that all coefficients are integers.

$\Rightarrow x+3y+14=0$ — Rearrange to the correct form.

Example 7

Decide whether or not each line is parallel or perpendicular to the line $y=4x-1$

a $2x+8y=5$ **b** $20x+5y=2$ **c** $16x-4y=5$

First note that the gradient of $y=4x-1$ is 4

a $2x+8y=5 \Rightarrow 8y=5-2x$ — Rearrange to make y the subject.

$\Rightarrow y=\frac{5}{8}-\frac{1}{4}x$ — The gradient is $-\frac{1}{4}$

$4\times\left(-\frac{1}{4}\right)=-1$ so this line is perpendicular to $y=4x-1$ — Since the product of the gradients is –1

b $20x+5y=2 \Rightarrow 5y=2-20x$

$\Rightarrow y=\frac{2}{5}-4x$ — Rearrange to make y the subject.

The gradient is –4 so this line is neither parallel nor perpendicular to $y=4x-1$

c $16x-4y=5 \Rightarrow 4y=16x-5$

$\Rightarrow y=4x-\frac{5}{4}$

The gradient is 4 so this line is parallel to $y=4x-1$

Example 8

The line l_1 has equation $7x+4y=8$ The line l_2 is perpendicular to l_1 and passes through the point (7, 3). Find the equation of l_2 in the form $ax+by+c=0$ where a, b and c are integers.

$l_1: 7x+4y=8 \Rightarrow 4y=-7x+8$

$\Rightarrow y=-\frac{7}{4}x+2$ — Rearrange to make y the subject so you can see what the gradient is.

So the gradient of l_1 is $-\frac{7}{4}$ and the gradient of l_2 is $\frac{4}{7}$ — Since $\left(-\frac{7}{4}\right)\times\frac{4}{7}=-1$

So the equation of l_2 is $y-3=\frac{4}{7}(x-7)$ — Use $y-y_1=m(x-x_1)$ to write the equation of l_2

$\Rightarrow 7y-21=4(x-7)$ — Multiply both sides by 7 so that all coefficients are integers.

$\Rightarrow 7y-21=4x-28$

$\Rightarrow 4x-7y-7=0$ — Rearrange to the correct form.

Example 9

Find the equation of the perpendicular bisector of the line segment joining (3, –4) and (9, –6)

Midpoint is $\left(\frac{3+9}{2}, \frac{-4+(-6)}{2}\right) = (6, -5)$ — Use $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Gradient of line segment is $\frac{-6-(-4)}{9-3} = -\frac{2}{6} = -\frac{1}{3}$ — Use $m = \frac{y_2 - y_1}{x_2 - x_1}$

So the perpendicular bisector has gradient $m = 3$ — Since they are perpendicular and $3 \times \left(-\frac{1}{3}\right) = -1$

The equation of the perpendicular bisector is $y-(-5) = 3(x-6)$

or $y = 3x - 23$ — Use $y - y_1 = m(x - x_1)$

1 Find the gradient of the line through each pair of points.

a (3, 7) and (2, 8)

b (5, 2) and (–4, –6)

c (1.3, 4.7) and (2.6, –3.1)

d $\left(\frac{1}{2}, \frac{1}{3}\right)$ and $\left(\frac{3}{4}, \frac{2}{3}\right)$

e $(\sqrt{3}, 2)$ and $(2\sqrt{3}, 5)$

f $(3a, a)$ and $(a, 5a)$

2 Calculate the exact distance between each pair of points.

a (8, 4) and (1, 3)

b (– 3, 9) and (12, –7)

c (5.9 ,6.2) and (–8.1, 3.8)

d $\left(\frac{1}{5}, -\frac{1}{5}\right)$ and $\left(\frac{3}{5}, -\frac{4}{5}\right)$

e $(5, -3\sqrt{2})$ and $(2, \sqrt{2})$

f $(k, -3k)$ and $(2k, -6k)$

3 Find the coordinates of the midpoint of each pair of points.

a (3, 9) and (1, 7)

b (2, –4) and (–3, –9)

c (2.1, 3.5) and (6.3, –3.7)

d $\left(\frac{2}{3}, -\frac{1}{2}\right)$ and $\left(-\frac{5}{3}, -\frac{3}{2}\right)$

e $(6\sqrt{5}, 2\sqrt{5})$ and $(-\sqrt{5}, \sqrt{5})$

f $(m, 2n)$ and $(3m, -2n)$

4 Work out the gradient and the y-intercept of these lines.

a $y=7x-4$

b $y+2x=3$

c $x-y=4$

d $3x+2y=7$

e $5x-2y=9$

f $5y-3x=0$

g $x+6y+3=0$

h $3(y-2)=4(x-1)$

5 Find the equation of the line through each pair of points.

a (2, 5) and (0, 6)

b (1, –3) and (2, –5)

c (4, 4) and (7, –7)

d (8, –2) and (4, –3)

e (–3, –7) and (5, 9)

f $(\sqrt{2}, -\sqrt{2})$ and $(3\sqrt{2}, 4\sqrt{2})$

6 Which of these lines is either parallel or perpendicular to the line with equation $y=6x+5$?

a $2x+12y+3=0$

b $18x+3y=2$

c $3x-\frac{1}{2}y+5=0$

7 Which of these lines is either parallel or perpendicular to the line with equation $y=\frac{2}{3}x-4$?

a $24x+16y+3=0$

b $6x+9y+2=0$

c $2x-3y=7$

8 Which of these lines is either parallel or perpendicular to the line with equation $6x+12y=1$?

a $2y=5-x$

b $9x=18y+4$

c $10x-5y+3=0$

In questions **9–13**, give your answers in the form $ax+by+c=0$ where a, b and c are integers.

9 The line l_1 has equation $y=5x+1$

a Find the equation of the line l_2 which is parallel to l_1 and passes through (3, –3)

b Find the equation of the line l_2 which is perpendicular to l_1 and passes through (–4, 1)

10 The line l_1 has equation $y=3+\frac{1}{2}x$

a Find the equation of the line l_2 which is parallel to l_1 and passes through (–1, 5)

b Find the equation of the line l_2 which is perpendicular to l_1 and passes through (6, 2)

11 The line l_1 has equation $3x+y=9$

a Find the equation of the line l_2 which is parallel to l_1 and passes through (8, –2)

b Find the equation of the line l_2 which is perpendicular to l_1 and passes through (–1, –1)

12 The line l_1 has equation $6x+5y+2=0$

a Find the equation of the line l_2 which is parallel to l_1 and passes through (4, 0)

b Find the equation of the line l_2 which is perpendicular to l_1 and passes through (12, 3)

13 The line l_1 has equation $6x-2y=1$

a Find the equation of the line l_2 which is parallel to l_1 and passes through $\left(\frac{1}{2}, 1\right)$

b Find the equation of the line l_2 which is perpendicular to l_1 and passes through $\left(-1, -\frac{1}{2}\right)$

14 Find the equation of the perpendicular bisector of the line segment joining each pair of points.

a (5, –7) and (–3, 5)

b (–5, –9) and (5, 5)

c (–6, 2) and (4, 12)

d (2, –7) and (–1, 2)

The ‘ Careful Reading’ Introductory Task

Careful reading is the skill of closely reading a text for deep understanding. Many students find this difficult, but it is definitely a skill you can learn. You may ask yourself the relevance of this within A Level maths but with the increasing length of questions sometimes worth 10 to 12 marks full understanding of the scenario, information and question is vital.

When reading some text, ask yourself: how many times did I read the text? Did I reread the more difficult sections? Did I summarise the key information?

TASK: Follow the instructions in the grid and then complete the reading task that follows

CAREFUL READING SKILL SHEET
1. READ FOR THE GIST Read the text twice to get the basic idea. Pay special attention to titles, diagrams and the first and last paragraphs.
2. HELP I DON'T UNDERSTAND THIS BIT! Read the tricky scetions few times. Use a dictionary to check meanings. If you still don't get it, mark the section with a question mark, and continue reading. Then go back to it when you have read the whole text.
3. UNDERLINE THE MAIN POINTS: Try to underline about 10 per cent or less, 20 per cent maximum. Use a pencil so you can rub out underlinings.
4. SUMMARISE WITH KEY POINTS OR A MIND-MAP OR OTHER SUITABLE GRAPHIC Read back over the underlined bits to ensure you have all the really important bits in your summary.
NOW YOU WILL UNDERSTAND THE TEXT MUCH BETTER!

Read the article on the link below and *t*hen create a mind map of the key themes and findings of the article. https://plus.maths.org/content/easy-pi

Easy as pi?

Here are a few suggested puzzles to get you thinking mathematically. You can find a further selection here:

https://nrich.maths.org/8767

The Root of the Problem

Age 14 to 18 ★★

Alison has been exploring sums with surds. She used a spreadsheet to make columns for square roots, and then added together various combinations.

Here is one of the sums she worked out:

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\ldots+\frac{1}{\sqrt{99}+\sqrt{100}}.$$

The answer surprised her!
Can you find a way to evaluate the sum without using a calculator or spreadsheet?

Click here for a hint: Show

Can you find other similar sums with surds that give whole number answers?

suggested solutions: https://nrich.maths.org/901/solution

Power Stack

Age 16 to 18 Short ★

Kimberly wants to define 3^{3^3} as $(3^3)^3$ but Nermeen thinks that such a stack of powers should be defined as $3^{(3^3)}$.

Do their definitions lead to the same numerical value? Is the same true if 3 is replaced with some other number?

How would Kimberly's and Nermeen's definitions most naturally extend to the definition of $3^{3^{3^3}}$? Do their definitions lead to the same numerical value? Is the same true if 3 is replaced with some other number?

Extension: Try to compute the approximate size of the numbers as powers of 10.

suggested solutions: https://nrich.maths.org/7039/solution

Symmetricality

Five equations and five unknowns. Is there an easy way to find the unknown values?

Age: 14 to 18

Challenge level: ★☆☆

Secondary
↳ ALGEBRA
↳ Equations & Formulae

Here is a set of five equations:

$$b+c+d+e=4$$
$$a+c+d+e=5$$
$$a+b+d+e=1$$
$$a+b+c+e=2$$
$$a+b+c+d=0$$

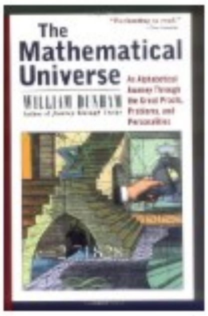
What do you notice when you add the five equations?

Can you now find the values of a, b, c, d and e?

suggested solutions: https://nrich.maths.org/problems/symmetricality?tab=solutions

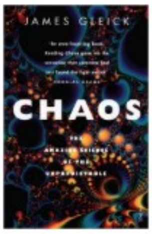
Just for fun- Additional suggested reading if you are interested in Maths.

The Mathematical Universe: Alphabetical Journey Through the Great Proofs, Problems & Personalities by William Dunham



In this book, Dunham takes us through a tantalizing selection of the great proofs, notorious disputes, and intriguing unsolved mysteries of the mathematical universe. Subjects range from the golden age of Greek geometry to the furthest frontier of infinite series. Dunham explores more than five thousand years of mathematical history, digging into the earliest records in Egypt, Babylon, India, and China, and turning up surprising tales and tidbits from modern times. All along the way, Dunham portrays the great masters of mathematics at their work. In colorful anecdotes, the brilliant - often eccentric - luminaries chart the course of mathematical progress. This book is accessible to any reader with a basic knowledge of algebra and geometry. You will come away from this exhilarating book with a keen sense of the power and splendor of the magical mathematical world.

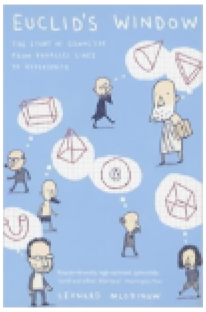
Chaos by James Gleick



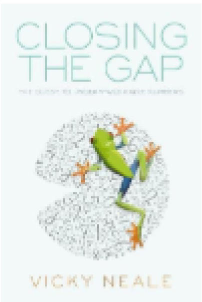
Chaos is what happens when the behaviour of a system gets too complicated to predict; the most familiar example is the weather, which apparently cannot be forecast accurately more than five days ahead. This book tells the story so far in the study of this new field of Physics.

Euclid's Window: The Story of Geometry from Parallel Lines to Hyperspace by Leonard Mlodinow

Anyone who thought geometry was boring or dry should prepare to be amazed. Despite its worthy cover this book is exactly what its title says - a story - and the plot of this story involves life, death and revolutions of understanding and belief. It stars the some of the most famous names in history, from Euclid who laid the logical foundations, to Albert Einstein, who united space and time in a single non-Euclidean geometry. It offers an alternative history of mathematics, revealing how simple questions anyone might ask about space - in the living room or in some other galaxy - have been the hidden engines of the highest achievements in science and technology.



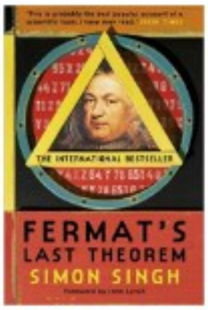
Closing the Gap: The Quest to Understand Prime Numbers by Vicky Neale



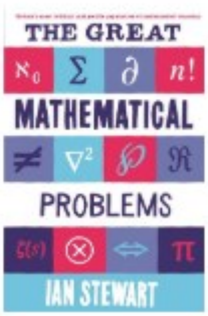
Prime numbers have intrigued, inspired and infuriated mathematicians for millennia. Every school student studies prime numbers and can appreciate their beauty, and yet mathematicians' difficulty with answering some seemingly simple questions about them reveals the depth and subtlety of prime numbers. In this book, Vicky Neale charts the recent progress towards proving the famous Twin Primes Conjecture, and the very different ways in which the breakthroughs have been made: a solo mathematician working in isolation and obscurity, and a large collaboration that is more public than any previous collaborative effort in mathematics. Interleaved with this story are highlights from a significantly older tale, going back two thousand years and more, of mathematicians' efforts to comprehend the beauty and unlock the mysteries of the prime numbers.

Fermat's Last Theorem by Simon Singh

The story of the solving of a puzzle that has confounded mathematicians since the 17th century. In 1963, a schoolboy browsing in his local library stumbled across the world's greatest mathematical problem: Fermat's Last Theorem, a puzzle that every child can understand but which has baffled mathematicians for over 300 years. Aged just ten, Andrew Wiles dreamed that he would crack it. Wiles's lifelong obsession with a seemingly simple challenge set by a long-dead Frenchman is an emotional tale of sacrifice and extraordinary determination. In the end, Wiles was forced to work in secrecy and isolation for seven years, harnessing all the power of modern maths to achieve his childhood dream. Many before him had tried and failed, including a 18-century philanderer who was killed in a duel. An 18-century Frenchwoman made a major breakthrough in solving the riddle, but she had to attend maths lectures at the Ecole Polytechnique disguised as a man since women were forbidden entry to the school.



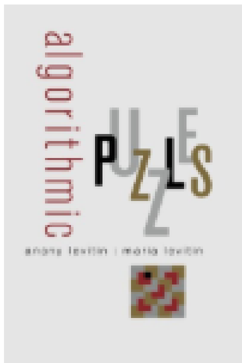
The Great Mathematical Problems by Ian Stewart



There are some mathematical problems whose significance goes beyond the ordinary - like Fermat's Last Theorem or Goldbach's Conjecture - they are the enigmas which define mathematics. This book explains why these problems exist, why they matter, what drives mathematicians to incredible lengths to solve them and where they stand in the context of mathematics and science as a whole. It contains solved problems - like the Poincaré Conjecture, cracked by the eccentric genius Grigori Perelman, who refused academic honours and a million-dollar prize for his work, and problems which, like the Riemann Hypothesis, remain baffling after centuries. Stewart is the guide to this mysterious and exciting world, showing how modern mathematicians constantly rise to the challenges set by their predecessors, as the great mathematical problems of the past succumb to the new techniques and ideas of the present.

Algorithmic Puzzles by Anany & Maria Levitin

In this book, Anany and Maria Levitin use many classic brainteasers as well as newer examples from job interviews with major corporations to show readers how to apply analytical thinking to solve puzzles requiring well-defined procedures. The book's unique collection of puzzles is supplemented with carefully developed tutorials on algorithm design strategies and analysis techniques intended to walk the reader step-by-step through the various approaches to algorithmic problem solving. Mastery of these strategies - exhaustive search, backtracking, and divide-and-conquer, among others - will aid the reader in solving not only the puzzles contained in this book, but also others encountered in interviews, puzzle collections, and throughout everyday life. Each of the 150 puzzles contains hints and solutions, along with commentary on the puzzle's origins and solution methods. Readers with only middle school mathematics will develop their algorithmic problem-solving skills through puzzles at the elementary level, while seasoned puzzle solvers will enjoy the challenge of thinking through more difficult puzzles.



Bridging Exercise 1A

1a $49^{\frac{1}{2}}=\sqrt{49}$
$=7$

1b $27^{\frac{1}{3}}=\sqrt[3]{27}$
$=3$

1c $5^{-1}=\frac{1}{5}$

1d $64^{-\frac{1}{3}}=\frac{1}{64^{\frac{1}{3}}}$
$=\frac{1}{\sqrt[3]{64}}$
$=\frac{1}{4}$

1e $9^{\frac{3}{2}}=\left(9^{\frac{1}{2}}\right)^3$
$=\left(\sqrt{9}\right)^3$
$=3^3$
$=27$

1f $16^{\frac{3}{4}}=\left(16^{\frac{1}{4}}\right)^3$
$=\left(\sqrt[4]{16}\right)^3$
$=2^3$
$=8$

1g $125^{-\frac{2}{3}}=\frac{1}{\left(125^{\frac{1}{3}}\right)^2}$
$=\frac{1}{\left(\sqrt[3]{125}\right)^2}$
$=\frac{1}{5^2}$
$=\frac{1}{25}$

1h $\left(\frac{1}{2}\right)^3=\frac{1^3}{2^3}$
$=\frac{1}{8}$

1i $\left(\frac{1}{9}\right)^{-2}=9^2$
$=81$

1j $\left(\frac{4}{9}\right)^{\frac{1}{2}}=\frac{4^{\frac{1}{2}}}{9^{\frac{1}{2}}}$
$=\frac{\sqrt{4}}{\sqrt{9}}$
$=\frac{2}{3}$

1k $\left(\frac{9}{16}\right)^{-0.5}=\left(\frac{16}{9}\right)^{0.5}$
$=\frac{16^{0.5}}{9^{0.5}}$
$=\frac{\sqrt{16}}{\sqrt{9}}$
$=\frac{4}{3}$

1l $\left(\frac{27}{8}\right)^{-\frac{2}{3}}=\left(\frac{8}{27}\right)^{\frac{2}{3}}$
$=\left(\sqrt[3]{\frac{8}{27}}\right)^2$
$=\left(\frac{\sqrt[3]{8}}{\sqrt[3]{27}}\right)^2$
$=\left(\frac{2}{3}\right)^2$
$=\frac{2^2}{3^2}$
$=\frac{4}{9}$

2a $\sqrt{8}=\sqrt{4}\sqrt{2}$
$=2\sqrt{2}$

2b $\sqrt{75}=\sqrt{25}\sqrt{3}$
$=5\sqrt{3}$

2c $2\sqrt{24}=2\sqrt{4}\sqrt{6}$
$=2\times2\sqrt{6}$
$=4\sqrt{6}$

2d $3\sqrt{48}=3\sqrt{16}\sqrt{3}$
$=3\times4\sqrt{3}$
$=12\sqrt{3}$

2e $\sqrt{20}+\sqrt{5}=\sqrt{4}\sqrt{5}+\sqrt{5}$
$=2\sqrt{5}+\sqrt{5}$
$=3\sqrt{5}$

2f $\sqrt{27}-\sqrt{12}=\sqrt{9}\sqrt{3}-\sqrt{4}\sqrt{3}$
$=3\sqrt{3}-2\sqrt{3}$
$=\sqrt{3}$

2g $5\sqrt{32}-3\sqrt{8}=5\sqrt{16}\sqrt{2}-3\sqrt{4}\sqrt{2}$
$=5\times4\sqrt{2}-3\times2\sqrt{2}$
$=20\sqrt{2}-6\sqrt{2}$
$=14\sqrt{2}$

2h $\sqrt{50}+3\sqrt{125}=\sqrt{25}\sqrt{2}+3\sqrt{25}\sqrt{5}$
$=5\sqrt{2}+15\sqrt{5}$

2i $\sqrt{68}+3\sqrt{17}=\sqrt{4}\sqrt{17}+3\sqrt{17}$
$=2\sqrt{17}+3\sqrt{17}$
$=5\sqrt{17}$

2j $3\sqrt{72}-\sqrt{32}=3\sqrt{36}\sqrt{2}-\sqrt{16}\sqrt{2}$
$=3\times6\sqrt{2}-4\sqrt{2}$
$=18\sqrt{2}-4\sqrt{2}$
$=14\sqrt{2}$

2k $4\sqrt{18}-2\sqrt{3}=4\sqrt{9}\sqrt{2}-2\sqrt{3}$
$=4\times3\sqrt{2}-2\sqrt{3}$
$=12\sqrt{2}-2\sqrt{3}$

2l $6\sqrt{5}+\sqrt{50}=6\sqrt{5}+\sqrt{25}\sqrt{2}$
$=6\sqrt{5}+5\sqrt{2}$

3a $\dfrac{1}{\sqrt{7}}=\dfrac{\sqrt{7}}{\sqrt{7}\sqrt{7}}$

$=\dfrac{\sqrt{7}}{7}$

3b $\dfrac{2}{\sqrt{8}}=\dfrac{2}{\sqrt{4}\sqrt{2}}$

$=\dfrac{2}{2\sqrt{2}}$

$=\dfrac{2\sqrt{2}}{2\sqrt{2}\sqrt{2}}$

$=\dfrac{\sqrt{2}}{2}$

3c $\dfrac{12}{\sqrt{3}}=\dfrac{12\sqrt{3}}{\sqrt{3}\sqrt{3}}$

$=\dfrac{12\sqrt{3}}{3}$

$=4\sqrt{3}$

3d $\dfrac{\sqrt{8}}{\sqrt{12}}=\dfrac{\sqrt{4}\sqrt{2}}{\sqrt{4}\sqrt{3}}$

$=\dfrac{2\sqrt{2}}{2\sqrt{3}}$

$=\dfrac{\sqrt{2}}{\sqrt{3}}$

$=\dfrac{\sqrt{2}\sqrt{3}}{\sqrt{3}\sqrt{3}}$

$=\dfrac{\sqrt{6}}{3}$

3e $\dfrac{1}{1+\sqrt{3}}=\dfrac{1-\sqrt{3}}{(1+\sqrt{3})(1-\sqrt{3})}$

$=\dfrac{1-\sqrt{3}}{1-\sqrt{3}+\sqrt{3}-3}$

$=\dfrac{1-\sqrt{3}}{-2}$

$=\dfrac{1}{2}(\sqrt{3}-1)$

3f $\dfrac{2}{1+\sqrt{2}}=\dfrac{2(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})}$

$=\dfrac{2(1-\sqrt{2})}{1-\sqrt{2}+\sqrt{2}-2}$

$=\dfrac{2(1-\sqrt{2})}{-1}$

$=-2(1-\sqrt{2})$

$=2(\sqrt{2}-1)$

3g $\dfrac{8}{1-\sqrt{5}}=\dfrac{8(1+\sqrt{5})}{(1-\sqrt{5})(1+\sqrt{5})}$

$=\dfrac{8(1+\sqrt{5})}{1+\sqrt{5}-\sqrt{5}-5}$

$=\dfrac{8(1+\sqrt{5})}{-4}$

$=-2(1+\sqrt{5})$

3h $\dfrac{2}{\sqrt{5}-1}=\dfrac{2(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)}$

$=\dfrac{2(\sqrt{5}+1)}{5+\sqrt{5}-\sqrt{5}-1}$

$=\dfrac{2(\sqrt{5}+1)}{4}$

$=\dfrac{1}{2}(\sqrt{5}+1)$

3i $\dfrac{\sqrt{2}}{2+\sqrt{3}}=\dfrac{\sqrt{2}(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$

$=\dfrac{\sqrt{2}(2-\sqrt{3})}{4-2\sqrt{3}+2\sqrt{3}-3}$

$=\dfrac{\sqrt{2}(2-\sqrt{3})}{1}$

$=2\sqrt{2}-\sqrt{6}$

3i $\dfrac{\sqrt{2}}{2+\sqrt{3}}=\dfrac{\sqrt{2}(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$

$=\dfrac{\sqrt{2}(2-\sqrt{3})}{4-2\sqrt{3}+2\sqrt{3}-3}$

$=\dfrac{\sqrt{2}(2-\sqrt{3})}{1}$

$=2\sqrt{2}-\sqrt{6}$

3j $\dfrac{2\sqrt{3}}{\sqrt{6}-2}=\dfrac{2\sqrt{3}(\sqrt{6}+2)}{(\sqrt{6}-2)(\sqrt{6}+2)}$

$=\dfrac{2\sqrt{3}(\sqrt{6}+2)}{6+2\sqrt{6}-2\sqrt{6}-4}$

$=\dfrac{2\sqrt{3}(\sqrt{6}+2)}{2}$

$=\sqrt{3}(\sqrt{6}+2)$

$=\sqrt{18}+2\sqrt{3}$

$=3\sqrt{2}+2\sqrt{3}$

3k $\dfrac{1+\sqrt{2}}{1-\sqrt{2}}=\dfrac{(1+\sqrt{2})(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})}$

$=\dfrac{1+\sqrt{2}+\sqrt{2}+2}{1+\sqrt{2}-\sqrt{2}-2}$

$=\dfrac{3+2\sqrt{2}}{-1}$

$=-3-2\sqrt{2}$

3l $\dfrac{3+\sqrt{5}}{\sqrt{5}-3}=\dfrac{(3+\sqrt{5})(\sqrt{5}+3)}{(\sqrt{5}-3)(\sqrt{5}+3)}$

$=\dfrac{3\sqrt{5}+9+5+3\sqrt{5}}{5+3\sqrt{5}-3\sqrt{5}-9}$

$=\dfrac{6\sqrt{5}+14}{-4}$

$=-\dfrac{3}{2}\sqrt{5}-\dfrac{7}{2}$

4a $(1+\sqrt{2})(3+\sqrt{2})=3+\sqrt{2}+3\sqrt{2}+2$
$=5+4\sqrt{2}$

4b $(1+\sqrt{2})(3-\sqrt{2})=3-\sqrt{2}+3\sqrt{2}-2$
$=1+2\sqrt{2}$

4c $(1-\sqrt{2})(3+\sqrt{2})=3+\sqrt{2}-3\sqrt{2}-2$
$=1-2\sqrt{2}$

4d $(1-\sqrt{2})(3-\sqrt{2})=3-\sqrt{2}-3\sqrt{2}+2$
$=5-4\sqrt{2}$

4e $(\sqrt{3}+2)(4+\sqrt{3})=4\sqrt{3}+3+8+2\sqrt{3}$
$=6\sqrt{3}+11$

4f $(\sqrt{3}+2)(4-\sqrt{3})=4\sqrt{3}-3+8-2\sqrt{3}$
$=2\sqrt{3}+5$

4g $(\sqrt{3}-2)(4+\sqrt{3})=4\sqrt{3}+3-8-2\sqrt{3}$
$=2\sqrt{3}-5$

4h $(\sqrt{3}-2)(4-\sqrt{3})=4\sqrt{3}-3-8+2\sqrt{3}$
$=6\sqrt{3}-11$

4i $(\sqrt{6}+1)(\sqrt{2}+3)=\sqrt{12}+3\sqrt{6}+\sqrt{2}+3$
$=2\sqrt{3}+3\sqrt{6}+\sqrt{2}+3$

4j $(\sqrt{6}+1)(\sqrt{2}-3)=\sqrt{12}-3\sqrt{6}+\sqrt{2}-3$
$=2\sqrt{3}-3\sqrt{6}+\sqrt{2}-3$

4k $(\sqrt{6}-1)(\sqrt{2}+3)=\sqrt{12}+3\sqrt{6}-\sqrt{2}-3$
$=2\sqrt{3}+3\sqrt{6}-\sqrt{2}-3$

4l $(\sqrt{6}-1)(\sqrt{2}-3)=\sqrt{12}-3\sqrt{6}-\sqrt{2}+3$
$=2\sqrt{3}-3\sqrt{6}-\sqrt{2}+3$

5a $x^3\times x^7=x^{3+7}$
$=x^{10}$

5b $7x^5\times 3x^6=7\times 3\times x^{5+6}$
$=21x^{11}$

5c $5x^4\times 8x^7=5\times 8\times x^{4+7}$
$=40x^{11}$

5d $x^8\div x^2=x^{8-2}$
$=x^6$

5e $8x^7\div 2x^9=\frac{8}{2}\times x^{7-9}$
$=4x^{-2}$

5f $3x^8\div 12x^7=\frac{3}{12}\times x^{8-7}$
$=\frac{1}{4}x$

5g $(x^5)^7=x^{5\times 7}$
$=x^{35}$

5h $(x^2)^{-5}=x^{2\times(-5)}$
$=x^{-10}$

5i $(3x^2)^4=3^4\times x^{2\times 4}$
$=81x^8$

5j $(6x^5)^2=6^2\times x^{5\times 2}$
$=36x^{10}$

5k $\sqrt{x^3}=(x^3)^{\frac{1}{2}}$
$=x^{\frac{3}{2}}$

5l $\sqrt[4]{x^5}=(x^5)^{\frac{1}{4}}$
$=x^{\frac{5}{4}}$

5m $\frac{5\sqrt{x}}{x}=\frac{5x^{\frac{1}{2}}}{x^1}$
$=5\times x^{\frac{1}{2}}x^{-1}$
$=5\times x^{\frac{1}{2}-1}$
$=5x^{-\frac{1}{2}}$

5n $2x\sqrt{x}=2\times x^1\times x^{\frac{1}{2}}$
$=2\times x^{1+\frac{1}{2}}$
$=2x^{\frac{3}{2}}$

5o $\frac{x^2}{3\sqrt{x}}=\frac{x^3}{3x^{\frac{1}{2}}}$
$=\frac{1}{3}\times x^{3-\frac{1}{2}}$
$=\frac{1}{3}x^{\frac{3}{2}}$

5p $x^3(x^5-1)=x^{3+5}-x^3$
$=x^8-x^3$

5q $x^3(\sqrt{x}+2)=x^3(x^{\frac{1}{2}}+$
$=x^{3+\frac{1}{2}}+2$
$=x^{\frac{7}{2}}+2x^3$

5r $\frac{x+2}{x^3}=\frac{x}{x^3}+\frac{2}{x^3}$
$=x^{1-3}+2x^{-3}$
$=x^{-2}+2x^{-3}$

5s $\frac{\sqrt{x}+3}{x}=\frac{x^{\frac{1}{2}}+3}{x^1}$
$=\frac{x^{\frac{1}{2}}}{x^1}+\frac{3}{x^1}$
$=x^{\frac{1}{2}-1}+3x^{-1}$
$=x^{-\frac{1}{2}}+3x^{-1}$

5t $\frac{3-x^3}{\sqrt{x}}=\frac{3-x^3}{x^{\frac{1}{2}}}$
$=\frac{3}{x^{\frac{1}{2}}}-\frac{x^3}{x^{\frac{1}{2}}}$
$=3x^{-\frac{1}{2}}-x^{3-\frac{1}{2}}$
$=3x^{-\frac{1}{2}}-x^{\frac{5}{2}}$

5u $(\sqrt{x}+3)^2=x+3\sqrt{x}+3\sqrt{x}+9$
$=x+6\sqrt{x}+9$

5v $\frac{3+\sqrt{x}}{x^2}=\frac{3}{x^2}+\frac{x^{\frac{1}{2}}}{x^2}$
$=3x^{-2}+x^{\frac{1}{2}-2}$
$=3x^{-2}+x^{-\frac{3}{2}}$

5w $\frac{1-x}{2\sqrt{x}}=\frac{1}{2x^{\frac{1}{2}}}-\frac{x}{2x^{\frac{1}{2}}}$
$=\frac{1}{2}\times\frac{1}{x^{\frac{1}{2}}}-\frac{1}{2}\times x^{1-\frac{1}{2}}$
$=\frac{1}{2}x^{-\frac{1}{2}}-\frac{1}{2}x^{\frac{1}{2}}$

5x $\frac{\sqrt{x}+2}{3x^3}=\frac{x^{\frac{1}{2}}}{3x^3}+\frac{2}{3x^3}$
$=\frac{1}{3}\times x^{\frac{1}{2}-3}+\frac{2}{3}\times x^{-3}$
$=\frac{1}{3}x^{-\frac{5}{2}}+\frac{2}{3}x^{-3}$

Bridging Exercise 1B

1a $3(2x+9)=7$

$6x+27=7$

$6x=-20$

$x=-\frac{20}{6}$

$=-\frac{10}{3}$

1b $7-3x=12$

$-3x=5$

$x=-\frac{5}{3}$

1c $\frac{x+4}{5}=7$

$x+4=35$

$x=31$

1d $2x+7=5x-6$

$7=3x-6$

$3x=13$

$x=\frac{13}{3}$

1e $8x-3=2(3x+1)$

$8x-3=6x+2$

$2x-3=2$

$2x=5$

$x=\frac{5}{2}$

$=2.5$

1f $\frac{2x+9}{12}=x-1$

$2x+9=12x-12$

$9=10x-12$

$10x=21$

$x=\frac{21}{10}$

$=2.1$

1g $2(3x-7)=4x$

$6x-14=4x$

$2x=14$

$x=7$

1h $7-2x=3(4-5x)$

$7-2x=12-15x$

$7-13x=12$

$13x=5$

$x=\frac{5}{13}$

2a $\frac{x}{2}+7\geq 5$

$\frac{x}{2}\geq -2$

$x\geq -4$

2b $3-4x<15$

$-4x<12$

$-12<4x$

$x>-3$

2c $5(x-1)>12+x$

$5x-5>12+x$

$4x>17$

$x>\frac{17}{4}$

2d $\frac{x+1}{3}>2$

$x+1>6$

$x>5$

2e $8x-1\leq 2x-5$

$6x-1\leq -5$

$6x\leq -4$

$x\leq -\frac{2}{3}$

2f $3(x+1)\geq \frac{x-3}{2}$

$6(x+1)\geq x-3$

$6x+6\geq x-3$

$5x+6\geq -3$

$5x\geq -9$

$x\geq -1.8$

2g $3(2x-5)<1-x$

$6x-15<1-x$

$7x-15<1$

$7x<16$

$x<\frac{16}{7}$

2h $x-(3+2x)\geq 2(x+1)$

$x-3-2x\geq 2x+2$

$-3-x\geq 2x+2$

$-3-3x\geq 2$

$-5\geq 3x$

$x\leq -\frac{5}{3}$

3a $2x+5=3A-1$

$2x=3A-6$

$x=\frac{3A-6}{2}$

3b $x+u=vx+3$

$x-vx=3-u$

$x(1-v)=3-u$

$x=\frac{3-u}{1-v}$

3c $\frac{3x-1}{k}=2x$

$3x-1=2kx$

$3x-2kx=1$

$x(3-2k)=1$

$x=\frac{1}{3-2k}$

3d $5(x-3m)=2nx-4$

$5x-15m=2nx-4$

$5x-2nx=15m-4$

$x(5-2n)=15m-4$

$x=\frac{15m-4}{5-2n}$

3e $(1-3x)^2=t$

$1-3x=\pm\sqrt{t}$

$-3x=-1\pm\sqrt{t}$

$3x=1\pm\sqrt{t}$

$x=\frac{1\pm\sqrt{t}}{3}$

3f $\frac{1}{x}=\frac{1}{p}+\frac{1}{q}$

$pq=px+qx$

$pq=x(p+q)$

$x=\frac{pq}{p+q}$

3g $\frac{1}{x^2+k}-6=4$

$\frac{1}{x^2+k}=10$

$x^2+k=\frac{1}{10}$

$x^2=\frac{1}{10}-k$

$x=\pm\sqrt{\frac{1}{10}-k}$

3h $\sqrt{x+A}=2B$

$x+A=(2B)^2$

$x+A=4B^2$

$x=4B^2-A$

4a $5x+12y=-6 \quad (1)$

$x+5y=4$

Multiply second equation by 5:

$5\times(x+5y)=5\times4$

$5x+25y=20 \quad (2)$

Then (2) – (1):

$(5x+25y)-(5x+12y)=20-(-6)$

$\Rightarrow 13y=26$

$\Rightarrow y=2$

Substitute y value into one of the original equations, for $x+5y=4$:

$x+5(2)=4$

$x+10=4$

$\Rightarrow x=-6$

4b $7x+5y=14,\ \ 3x+4y=19$

Multiply first equation by 4:

$4\times(7x+5y)=4\times14$

$28x+20y=56 \ (1)$

Multiply second equation by 5:

$5\times(3x+4y)=5\times19$

$15x+20y=95 \ (2)$

(1) – (2):

$(28x+20y)-(15x+20y)=56-95$

$\Rightarrow 13x=-39$

$\Rightarrow x=-3$

Substitute x value into one of the original equations, for $7x+5y=14$:

$7(-3)+5y=14$

$-21+5y=14$

$\Rightarrow y=7$

4c $2x-5y=4,\ \ 3x-8y=5$

Multiply first equation by 3:

$3\times(2x-5y)=3\times4$

$6x-15y=12 \quad (1)$

Multiply second equation by 2:

$2\times(3x-8y)=2\times5$

$6x-16y=10 \quad (2)$

(1) – (2):

$(6x-15y)-(6x-16y)=12-10$

$\Rightarrow y=2$

Substitute y value into one of the original equations, for $2x-5y=4$:

$2x-5(2)=4$

$2x-10=4$

$\Rightarrow x=7$

4d $3x-2y=2,\ \ 8x+3y=4.5$

Multiply first equation by 3:

$3\times(3x-2y)=3\times2$

$9x-6y=6 \quad (1)$

Multiply second equation by 2:

$2\times(8x+3y)=2\times4.5$

$16x+6y=9 \quad (2)$

(1)+ (2):

$(9x-6y)+(16x+6y)=6+9$

$\Rightarrow 25x=15$

$\Rightarrow x=\frac{3}{5}$

Substitute x value into one of the original equations, for $3x-2y=2$:

$\Rightarrow 3\left(\frac{3}{5}\right)-2y=2$

$\frac{9}{5}-2y=2$

$-2y=\frac{1}{5}$

$\Rightarrow y=-\frac{1}{10}$

4e $5x-2y=11,\ -2x+3y=22$

Multiply the first equation by 2:

$2\times(5x-2y)=2\times11$

$10x-4y=22 \quad (1)$

Multiply the second equation by 5:

$5\times(-2x+3y)=5\times22$

$-10x+15y=110 \ (2)$

(1) + (2):

$(10x-4y)+(-10x+15y)=22+110$

$\Rightarrow 11y=132$

$\Rightarrow y=12$

Substitute y value into one of the original equations, for $5x-2y=11$:

$\Rightarrow 5x-2(12)=11$

$5x-24=11$

$5x=35$

$\Rightarrow x=7$

4f $8x+5y=-0.5,\ -6x+4y=-3.5$

Multiply first equation by 3:

$$3\times(8x+5y)=3\times-0.5$$
$$24x+15y=-1.5 \qquad (1)$$

Multiply second equation by 4:

$$4\times(-6x+4y)=4\times-3.5$$
$$-24x+16y=-14 \qquad (2)$$

(1) + (2):

$$(24x+15y)+(-24x+16y)=-1.5+-14$$
$$\Rightarrow 31y=-15.5$$
$$\Rightarrow y=-\frac{1}{2}$$

Substitute y value into one of the original equations, for $8x+5y=-0.5$:

$$\Rightarrow 8x+5\left(-\frac{1}{2}\right)=-0.5$$
$$8x-\frac{5}{2}=-\frac{1}{2}$$
$$8x=2$$
$$\Rightarrow x=\frac{1}{4}$$

5a $y=8-3x,\ y=2-5x$

Substitute for y:

$$2-5x=8-3x$$
$$-6=2x$$
$$x=-3$$

Substitute x value into one of the original equations, for $y=8-3x$:

$$\Rightarrow y=8-3(-3)$$
$$=8+9$$
$$=17$$

So the lines intersect at $(-3, 17)$

5b $y=7x-4,\ y=3x-2$

Substitute for y:

$$3x-2=7x-4$$
$$2=4x$$
$$x=\frac{1}{2}$$

Substitute x value into one of the original equations, for $y=3x-2$:

$$\Rightarrow y=3\left(\frac{1}{2}\right)-2$$
$$=\frac{3}{2}-2$$
$$=-\frac{1}{2}$$

So the lines intersect at $\left(\frac{1}{2},-\frac{1}{2}\right)$

5c $y=2x+3,\ y=5-x$

Substitute for y:

$$5-x=2x+3$$
$$3x=2$$
$$x=\frac{2}{3}$$

Substitute x value into one of the original equations, for $y=5-x$:

$$\Rightarrow y=5-\frac{2}{3}$$
$$=\frac{13}{3}$$

So the lines intersect at $\left(\frac{2}{3},\frac{13}{3}\right)$

5d $y+5=3x,\ y=-5x+7$

Substitute for y:

$$3x-5=-5x+7$$
$$8x=12$$
$$x=1.5$$

Substitute x value into one of the original equations, for $y=-5x+7$:

$$\Rightarrow y=-5(1.5)+7$$
$$=-0.5$$

So the lines intersect at $(1.5, -0.5)$

5e $y=\frac{1}{2}x+3,\ y=5-2x$

Substitute for y:

$$\frac{1}{2}x+3=5-2x$$
$$\frac{5}{2}x=2$$
$$x=0.8$$

Substitute x value into one of the original equations, for $y=5-2x$:

$$\Rightarrow y=5-2(0.8)$$
$$=5-1.6$$
$$=3.4$$

So the lines intersect at $(0.8, 3.4)$

5f $y=3(x+2),\ y=7-2x$

Substitute for y:

$$3x+6=7-2x$$
$$5x=1$$
$$x=0.2$$

Substitute x value into one of the original equations, for $y=7-2x$:

$$\Rightarrow y=7-2(0.2)$$
$$=7-0.4$$
$$=6.6$$

So the lines intersect at $(0.2, 6.6)$

Bridging Exercise 1C

1a $3x^2+5x=x(3x+5)$

1b $8x^2-4x=4x(2x-1)$

1c $17x^2+34x=17x(x+2)$

1d $18x^2-24x=6x(3x-4)$

2a $x^2+5x+6=(x+2)(x+3)$

2b $x^2-7x+10=(x-5)(x-2)$

2c $x^2-5x-6=(x-6)(x+1)$

2d $x^2+3x-28=(x+7)(x-4)$

2e $x^2-x-72=(x-9)(x+8)$

2f $x^2+2x-48=(x+8)(x-6)$

2g $x^2-12x+11=(x-11)(x-1)$

2h $x^2-5x-24=(x-8)(x+3)$

3a $x^2-100=(x+10)(x-10)$

3b $x^2-81=(x+9)(x-9)$

3c $4x^2-9=(2x+3)(2x-3)$

3d $64-9x^2=(8+3x)(8-3x)$

4a $3x^2+7x+2=3x^2+6x+x+2$

$=3x(x+2)+(x+2)$

$=(3x+1)(x+2)$

4b $6x^2+17x+12=6x^2+9x+8x+12$

$=3x(2x+3)+4(2x+3)$

$=(3x+4)(2x+3)$

4c $4x^2-13x+3=4x^2-12x-x+3$

$=4x(x-3)-(x-3)$

$=(4x-1)(x-3)$

4d $2x^2-7x-15=2x^2-10x+3x-15$

$=2x(x-5)+3(x-5)$

$=(2x+3)(x-5)$

4e $2x^2+3x-5=2x^2-2x+5x-5$

$=2x(x-1)+5(x-1)$

$=(2x+5)(x-1)$

4f $7x^2+25x-12=7x^2+28x-3x-12$

$=7x(x+4)-3(x+4)$

$=(7x-3)(x+4)$

4g $8x^2-22x+15=8x^2-12x-10x+15$

$=4x(2x-3)-5(2x-3)$

$=(4x-5)(2x-3)$

4h $12x^2+17x-5=12x^2+20x-3x-5$

$=4x(3x+5)-(3x+5)$

$=(4x-1)(3x+5)$

5a $16x^2-25=(4x+5)(4x-5)$

5b $4x^2-16x=4x(x-4)$

5c $x^2+13x+12=(x+12)(x+1)$

5d $3x^2+16x-35=3x^2+21x-5x-35$

$=3x(x+7)-5(x+7)$

$=(3x-5)(x+7)$

5e $x^2+x-12=(x+4)(x-3)$

5f $100-9x^2=(10+3x)(10-3x)$

5g $2x^2-14x=2x(x-7)$

5h $20x^2-3x-2=20x^2+5x-8x-2$

$=5x(4x+1)-2(4x+1)$

$=(5x-2)(4x+1)$

6a $21x^2-7x=7x(3x-1)$

$7x(3x-1)=0$

$x=0$ or $x=\frac{1}{3}$

6b $x^2-36=(x+6)(x-6)$

$(x+6)(x-6)=0$

$x=-6$ or $x=6$

6c $17x^2+34x=17x(x+2)$

$17x(x+2)=0$

$x=0$ or $x=-2$

6d $6x^2+13x+5=6x^2+10x+3x+5$

$=2x(3x+5)+(3x+5)$

$=(2x+1)(3x+5)$

$(2x+1)(3x+5)=0$

$x=-\frac{1}{2}$ or $x=-\frac{5}{3}$

6e $4x^2-49=(2x+7)(2x-7)$

$(2x+7)(2x-7)=0$

$x=-\frac{7}{2}$ or $x=\frac{7}{2}$

6f $x^2-7x-18=0$

$x^2-7x-18=(x-9)(x+2)$

$(x-9)(x+2)=0$

$x=9$ or $x=-2$

6g $x^2-7x+6=(x-6)(x-1)$

$(x-6)(x-1)=0$

$x=6$ or $x=1$

6h $21x^2+x-2=0$

$$21x^2+x-2=21x^2+7x-6x-2$$
$$=7x(3x+1)-2(3x+1)$$
$$=(7x-2)(3x+1)$$
$$(7x-2)(3x+1)=0$$
$$x=\frac{2}{7} \quad \text{or} \quad x=-\frac{1}{3}$$

6i $5x^2-17x+6=0$

$$5x^2-17x+6=5x^2-15x-2x+6$$
$$=5x(x-3)-2(x-3)$$
$$=(5x-2)(x-3)$$
$$(5x-2)(x-3)=0$$
$$x=\frac{2}{5} \quad \text{or} \quad x=3$$

6j $16x^2+24x+9=16x^2+12x+12x+9$

$$=4x(4x+3)+3(4x+3)$$
$$=(4x+3)^2$$
$$(4x+3)^2=0$$
$$4x+3=0$$
$$x=-\frac{3}{4}$$

6k $9x^2-12x+4=0$

$$9x^2-12x+4=9x^2-6x-6x+4$$
$$=3x(3x-2)-2(3x-2)$$
$$=(3x-2)^2$$
$$(3x-2)^2=0$$
$$3x-2=0$$
$$x=\frac{2}{3}$$

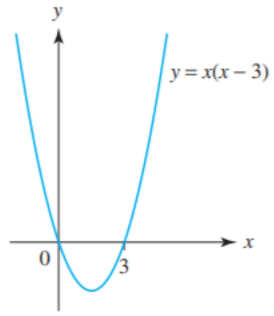
6l $40x^2+x-6=0$

$$40x^2+x-6=40x^2+16x-15x-6$$
$$=8x(5x+2)-3(5x+2)$$
$$=(8x-3)(5x+2)$$
$$(8x-3)(5x+2)=0$$
$$x=\frac{3}{8} \quad \text{or} \quad x=-\frac{2}{5}$$

7a $y=x(x-3)$

When $y=0$, $x(x-3)=0 \Rightarrow x=0$ or $x=3$

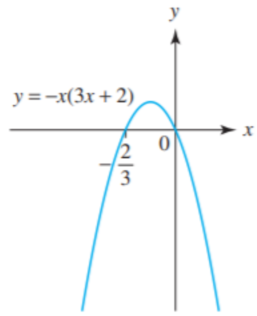
When $x=0$, $y=0$



7b $y=-x(3x+2)$

When $y=0$, $-x(3x+2)=0 \Rightarrow x=0$ or $x=-\frac{2}{3}$

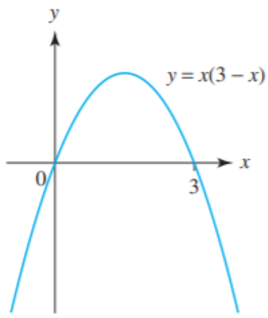
When $x=0$, $y=0$



7c $y=x(3-x)$

When $y=0$, $x(3-x)=0 \Rightarrow x=0$ or $x=3$

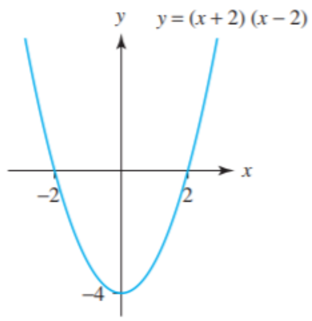
When $x=0$, $y=0$



7d $y=(x+2)(x-2)$

When $y=0$, $(x+2)(x-2)=0 \Rightarrow x=\pm 2$

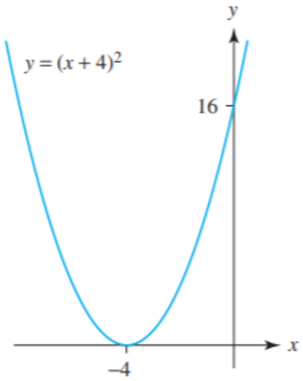
When $x=0$, $y=2\times -2=-4$



7e $y=(x+4)^2$

When $y=0$, $(x+4)^2=0 \Rightarrow x=-4$

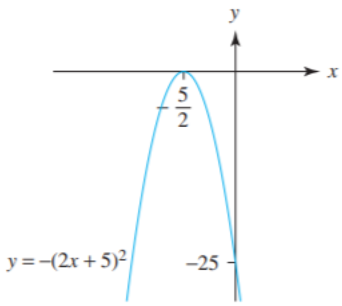
When $x=0$, $y=4^2=16$



7f $y=-(2x+5)^2$

When $y=0$, $-(2x+5)^2=0 \Rightarrow x=-\dfrac{5}{2}$

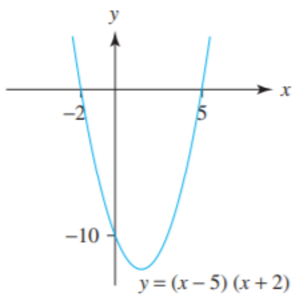
When $x=0$, $y=-5^2=-25$



7g $y=(x-5)(x+2)$

When $y=0$, $(x-5)(x+2)=0 \Rightarrow x=5$ or $x=-2$

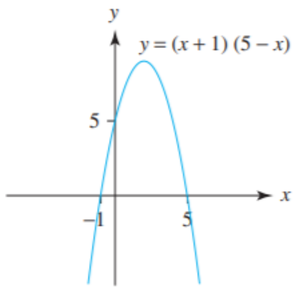
When $x=0$, $y=-5\times 2=-10$



7h $y=(x+1)(5-x)$

When $y=0$, $(x+1)(5-x)=0 \Rightarrow x=-1$ or $x=5$

When $x=0$, $y=1\times 5=5$

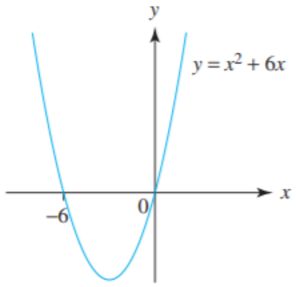


8a $y=x^2+6x$

$x^2+6x=x(x+6)$

When $y=0$, $x(x+6)=0 \Rightarrow x=0$ or $x=-6$

When $x=0$, $y=0$

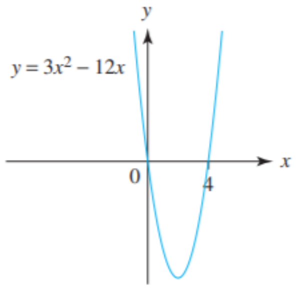


8b $y=3x^2-12x$

$3x^2-12x=3x(x-4)$

When $y=0$, $3x(x-4)=0 \Rightarrow x=0$ or $x=4$

When $x=0$, $y=0$

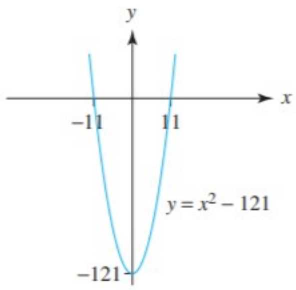


8c $y=x^2-121$

$x^2-121=(x+11)(x-11)$

When $y=0$, $(x+11)(x-11)=0 \Rightarrow x=\pm 11$

When $x=0$, $y=-121$

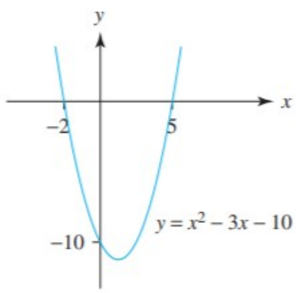


8d $y=x^2-3x-10$

$x^2-3x-10=(x-5)(x+2)$

When $y=0$, $(x-5)(x+2)=0 \Rightarrow x=5$ or $x=-2$

When $x=0$, $y=-10$

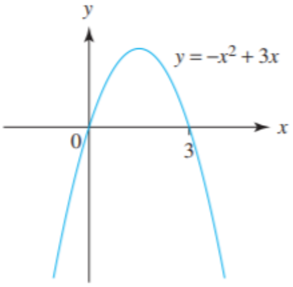


8e $y=-x^2+3x$

$-x^2+3x=-x(x-3)$

When $y=0$, $-x(x-3)=0 \Rightarrow x=0$ or $x=3$

When $x=0$, $y=0$

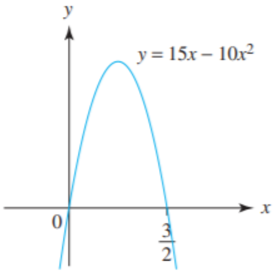


8f $y=15x-10x^2$

$15x-10x^2=5x(3-2x)$

When $y=0$, $5x(3-2x)=0 \Rightarrow x=0$ or $x=\frac{3}{2}$

When $x=0$, $y=0$

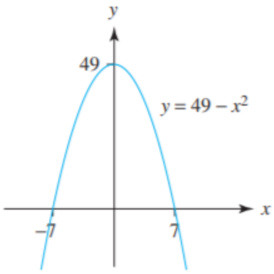


8g $y=49-x^2$

$49-x^2=(7-x)(x+7)$

When $y=0$, $(7-x)(7+x)=0 \Rightarrow x=\pm 7$

When $x=0$, $y=49$



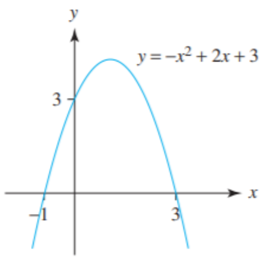
8h $y=-x^2+2x+3$

$-x^2+2x+3=-(x^2-2x-3)$

$=-(x-3)(x+1)$

When $y=0$, $-(x-3)(x+1)=0 \Rightarrow x=3$ or $x=-1$

When $x=0$, $y=3$

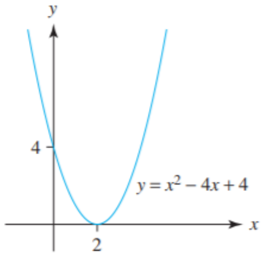


8i $y=x^2-4x+4$

$x^2-4x+4=(x-2)^2$

When $y=0$, $(x-2)^2=0 \Rightarrow x=2$

When $x=0$, $y=4$



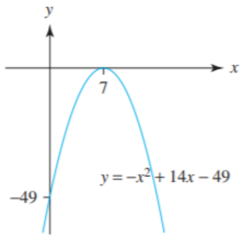
8j $y=-x^2+14x-49$

$-x^2+14x-49=-(x^2-14x+49)$

$=-(x-7)^2$

When $y=0$, $-(x-7)^2=0 \Rightarrow x=7$

When $x=0$, $y=-49$



8k $y=3x^2+4x+1$

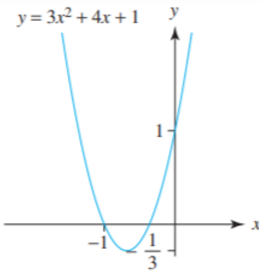
$3x^2+4x+1=3x^2+3x+x+1$

$=3x(x+1)+(x+1)$

$=(3x+1)(x+1)$

When $y=0$, $(3x+1)(x+1)=0 \Rightarrow x=-1$ or $x=-\frac{1}{3}$

When $x=0$, $y=1$



8l $y=-2x^2+11x-12$

$-2x^2+11x-12=-(2x^2-11x+12)$

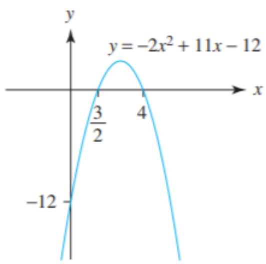
$=-(2x^2-8x-3x+12)$

$=-(2x(x-4)-3(x-4))$

$=-(2x-3)(x-4)$

When $y=0$, $-(2x-3)(x-4)=0 \Rightarrow x=\frac{3}{2}$ or $x=4$

When $x=0$, $y=-12$



Bridging Exercise 1D

1a $x^2+8x=(x+4)^2-16$

1b $x^2-18x=(x-9)^2-81$

1c $x^2+6x+3=(x+3)^2-9+3$

$=(x+3)^2-6$

1d $x^2+12x-5=(x+6)^2-6^2-5$

$=(x+6)^2-36-5$

$=(x+6)^2-41$

1e $x^2-7x+10=\left(x-\frac{7}{2}\right)^2-\frac{49}{4}+10$

$=\left(x-\frac{7}{2}\right)^2-\frac{9}{4}$

1f $x^2+5x+9=\left(x+\frac{5}{2}\right)^2-\frac{25}{4}+9$

$=\left(x+\frac{5}{2}\right)^2+\frac{11}{4}$

1g $2x^2+8x+4=2\left[x^2+4x+2\right]$

$=2\left[(x+2)^2-4+2\right]$

$=2\left[(x+2)^2-2\right]$

$=2(x+2)^2-4$

1h $3x^2+18x-6=3\left[x^2+6x-2\right]$

$=3\left[(x+3)^2-9-2\right]$

$=3\left[(x+3)^2-11\right]$

$=3(x+2)^2-33$

1i $2x^2-10x+3=2\left[x^2-5x+\frac{3}{2}\right]$

$=2\left[\left(x-\frac{5}{2}\right)^2-\frac{25}{4}+\frac{3}{2}\right]$

$=2\left[\left(x-\frac{5}{2}\right)^2-\frac{19}{4}\right]$

$=2\left(x-\frac{5}{2}\right)^2-\frac{19}{2}$

1j $-x^2+12x-1=-\left[x^2-12x+1\right]$

$=-\left[(x-6)^2-36+1\right]$

$=-\left[(x-6)^2-35\right]$

$=-(x-6)^2+35$

1k $-x^2+9x-3=-\left[x^2-9x+3\right]$

$=-\left[\left(x-\frac{9}{2}\right)^2-\frac{81}{4}+3\right]$

$=-\left[\left(x-\frac{9}{2}\right)^2-\frac{69}{4}\right]$

$=-\left(x-\frac{9}{2}\right)^2+\frac{69}{4}$

1l $-2x^2+5x-1=-2\left[x^2-\frac{5}{2}x+\frac{1}{2}\right]$

$=-2\left[\left(x-\frac{5}{4}\right)^2-\frac{25}{16}+\frac{1}{2}\right]$

$=-2\left[\left(x-\frac{5}{4}\right)^2-\frac{17}{16}\right]$

$=-2\left(x-\frac{5}{4}\right)^2+\frac{17}{8}$

2a $x^2+14x=(x+7)^2-49$

So $(-7, -49)$ is a minimum point

2b $x^2-18x+3=(x-9)^2-81+3$

$=(x-9)^2-78$

So $(9, -78)$ is a minimum point

2c $x^2-9x=\left(x-\frac{9}{2}\right)^2-\frac{81}{4}$

So $\left(\frac{9}{2},-\frac{81}{4}\right)$ is a minimum point

2d $-x^2+4x=-\left[x^2-4x\right]$

$=-\left[(x-2)^2-4\right]$

$=-(x-2)^2+4$

So $(2, 4)$ is a maximum point

2e $x^2+11x+30=\left(x+\frac{11}{2}\right)^2-\frac{121}{4}+30$

$=\left(x+\frac{11}{2}\right)^2-\frac{1}{4}$

So $\left(-\frac{11}{2},-\frac{1}{4}\right)$ is a minimum point

2f $-x^2+6x-7=-\left[x^2-6x+7\right]$

$=-\left[(x-3)^2-9+7\right]$

$=-\left[(x-3)^2-2\right]$

$=-(x-3)^2+2$

So $(3, 2)$ is a maximum point

2g $2x^2+16x-5=2\left[x^2+8x-\frac{5}{2}\right]$

$=2\left[(x+4)^2-16-\frac{5}{2}\right]$

$=2\left[(x+4)^2-\frac{37}{2}\right]$

$=2(x+4)^2-37$

So $(-4, -37)$ is a minimum point

2h $-3x^2+15x-2=-3\left[x^2-5x+\frac{2}{3}\right]$

$=-3\left[\left(x-\frac{5}{2}\right)^2-\frac{25}{4}+\frac{2}{3}\right]$

$=-3\left[\left(x-\frac{5}{2}\right)^2-\frac{67}{12}\right]$

$=-3\left(x-\frac{5}{2}\right)^2+\frac{67}{4}$

So $\left(\frac{5}{2},\frac{67}{4}\right)$ is a maximum point.

Bridging Exercise 1E

1a $7x^2+3x-8=0$

$a=7,\ b=3,\ c=-8$

$x=\dfrac{-3+\sqrt{3^2-4\times7\times(-8)}}{2\times7}$

$=0.88$

$x=\dfrac{-3-\sqrt{3^2-4\times7\times(-8)}}{2\times7}$

$=-1.30$

$x=0.88$ or $x=-1.30$

1b $-x^2+4x-2=0$

$a=-1,\ b=4,\ c=-2$

$x=\dfrac{-4+\sqrt{4^2-4\times(-1)\times(-2)}}{2\times(-1)}$

$=3.41$

$x=\dfrac{-4-\sqrt{4^2-4\times(-1)\times(-2)}}{2\times(-1)}$

$=0.59$

$x=3.41$ or $x=0.59$

1c $x^2-12x+4=0$

$a=1,\ b=-12,\ c=4$

$x=\dfrac{-(-12)+\sqrt{(-12)^2-4\times1\times4}}{2\times1}$

$=11.66$

$x=\dfrac{-(-12)-\sqrt{(-12)^2-4\times1\times4}}{2\times1}$

$=0.34$

$x=11.66$ or $x=0.34$

Bridging Exercise 1F

1a $m=\dfrac{8-7}{2-3}$

$=\dfrac{1}{-1}$

$=-1$

1b $m=\dfrac{-6--2}{-4-5}$

$=-\dfrac{4}{-9}$

$=\dfrac{4}{9}$

1c $m=\dfrac{-3.1-4.7}{2.6-1.3}$

$=-\dfrac{7.8}{1.3}$

$=-6$

1d $m=\dfrac{\frac{2}{3}-\frac{1}{3}}{\frac{3}{4}-\frac{1}{2}}$

$=\dfrac{\frac{1}{3}}{\frac{1}{4}}$

$=\dfrac{4}{3}$

1e $m=\dfrac{5-2}{2\sqrt{3}-\sqrt{3}}$

$=\dfrac{3}{\sqrt{3}}$

$=\sqrt{3}$

1f $m=\dfrac{5a-a}{a-3a}$

$=\dfrac{4a}{-2a}$

$=-2$

2a $d=\sqrt{(1-8)^2+(3-4)^2}$

$=\sqrt{(-7)^2+(-1)^2}$

$=5\sqrt{2}$

2b $d=\sqrt{(12--3)^2+(-7-9)^2}$

$=\sqrt{15^2+(-16)^2}$

$=\sqrt{481}$

2c $d=\sqrt{(-8.1-5.9)^2+(3.8-6.2)^2}$

$=\sqrt{14^2+(-2.4)^2}$

$=\sqrt{201.76}$

$=14.2$

2d $d=\sqrt{\left(\dfrac{3}{5}-\dfrac{1}{5}\right)^2+\left(-\dfrac{4}{5}--\dfrac{1}{5}\right)^2}$

$=\sqrt{\left(\dfrac{2}{5}\right)^2+\left(-\dfrac{3}{5}\right)^2}$

$=\dfrac{\sqrt{13}}{5}$

2e $d=\sqrt{(2-5)^2+(\sqrt{2}--3\sqrt{2})^2}$

$=\sqrt{3^2+(4\sqrt{2})^2}$

$=\sqrt{41}$

2f $d=\sqrt{(2k-k)^2+(-6k--3k)^2}$
$=\sqrt{k^2+(-3k)^2}$
$=k\sqrt{10}$

3a $\left(\frac{3+1}{2},\frac{9+7}{2}\right)=(2, 8)$

3b $\left(\frac{2+-3}{2},\frac{-4+-9}{2}\right)=(-0.5, -6.5)$

3c $\left(\frac{2.1+6.3}{2},\frac{3.5+-3.7}{2}\right)=(4.2, -0.1)$

3d $\left(\frac{\frac{2}{3}+-\frac{5}{3}}{2},\frac{-\frac{1}{2}+-\frac{3}{2}}{2}\right)=(-0.5, -1)$

3e $\left(\frac{6\sqrt{5}+-\sqrt{5}}{2},\frac{2\sqrt{5}+\sqrt{5}}{2}\right)=\left(\frac{5}{2}\sqrt{5},\frac{3}{2}\sqrt{5}\right)$

3f $\left(\frac{m+3m}{2},\frac{2n+-2n}{2}\right)=(2m, 0)$

4a $y=7x-4$
Gradient is 7, y-intercept is -4

4b Rearrange $y+2x=3$
$y=-2x+3$
Gradient is -2, y-intercept is 3

4c Rearrange $x-y=4$
$y=x-4$
Gradient is 1, y-intercept is -4

4d Rearrange $3x+2y=7$
$2y=7-3x$
$y=\frac{7}{2}-\frac{3}{2}x$
Gradient is $-\frac{3}{2}$, y-intercept is $\frac{7}{2}$

4e Rearrange $5x-2y=9$
$2y=5x-9$
$y=\frac{5}{2}x-\frac{9}{2}$
Gradient is $\frac{5}{2}$, y-intercept is $-\frac{9}{2}$

4f Rearrange $5y-3x=0$
$5y=3x$
$x=\frac{3}{5}x$
Gradient is $\frac{3}{5}$, y-intercept is 0

4g Rearrange $x+6y+3=0$
$6y=-3-x$
$y=-\frac{1}{2}-\frac{1}{6}x$
Gradient is $-\frac{1}{6}$, y-intercept is $-\frac{1}{2}$

4h Expand brackets and rearrange
$3(y-2)=4(x-1)$
$3y-6=4x-4$
$3y=4x+2$
$y=\frac{4}{3}x+\frac{2}{3}$
Gradient is $\frac{4}{3}$, y-intercept is $\frac{2}{3}$

5a $m=\frac{6-5}{0-2}$
$=\frac{1}{-2}$
$=-\frac{1}{2}$
$y-5=-\frac{1}{2}(x-2)$ or $y-6=-\frac{1}{2}(x-0)$
$y=-\frac{1}{2}x+6$

5b $m=\frac{-5--3}{2-1}$
$=\frac{-2}{1}$
$=-2$
$y+3=-2(x-1)$ or $y+5=-2(x-2)$
$y=-2x-1$

5c $m=\frac{-7-4}{7-4}$
$=-\frac{11}{3}$
$y-4=-\frac{11}{3}(x-4)$ or $y+7=-\frac{11}{3}(x-7)$
$11x+3y-56=0$

5d $m=\frac{-3--2}{4-8}$
$=\frac{-1}{-4}$
$=\frac{1}{4}$
$y+2=\frac{1}{4}(x-8)$ or $y+3=\frac{1}{4}(x-4)$
$y=\frac{1}{4}x-4$

5e $m=\frac{9--7}{5--3}$

$=\frac{16}{8}$

$=2$

$y+7=2(x+3)$ or $y-9=2(x-5)$

$y=2x-1$

5f $m=\frac{4\sqrt{2}--\sqrt{2}}{3\sqrt{2}-\sqrt{2}}$

$=\frac{5\sqrt{2}}{2\sqrt{2}}$

$=\frac{5}{2}$

$y+\sqrt{2}=\frac{5}{2}(x-\sqrt{2})$ or $y-4\sqrt{2}=\frac{5}{2}(x-3\sqrt{2})$

$y=\frac{5}{2}x-\frac{7}{2}\sqrt{2}$

6a $12y=-2x-3$

$y=-\frac{1}{6}x-\frac{1}{4}$

Gradient is $-\frac{1}{6}$ so perpendicular to $y=6x+5$ since $\left(-\frac{1}{6}\right)\times 6=-1$

6b $3y=2-18x$

$y=\frac{2}{3}-6x$

Gradient is -6 so neither parallel nor perpendicular to $y=6x+5$

6c $\frac{1}{2}y=3x+5$

$y=6x+10$

Gradient is 6 so parallel to $y=6x+5$

7a $16y=-24x-3$

$y=-\frac{3}{2}x-\frac{3}{16}$

Gradient is $-\frac{3}{2}$ so perpendicular to $y=\frac{2}{3}x-4$ since $\left(-\frac{3}{2}\right)\times\frac{2}{3}=-1$

7b $9y=-6x-2$

$y=-\frac{2}{3}x-\frac{2}{9}$

Gradient is $-\frac{2}{3}$ so neither parallel nor perpendicular to $y=\frac{2}{3}x-4$

7c $3y=2x-7$

$y=\frac{2}{3}x-\frac{7}{3}$

Gradient is $\frac{2}{3}$ so parallel to $y=\frac{2}{3}x-\frac{7}{3}$

8 Rearrange $12y=1-6x$

$y=\frac{1}{12}-\frac{1}{2}x$

so gradient is $-\frac{1}{2}$

8a Rearrange $2y=5-x$

$y=\frac{5}{2}-\frac{1}{2}x$

Gradient is $-\frac{1}{2}$ so parallel to $6x+12y=1$

8b Rearrange $9x=18y+4$

$18y=9x-4$

$y=\frac{1}{2}x-\frac{2}{9}$

Gradient is $\frac{1}{2}$ so neither parallel nor perpendicular to $6x+12y=1$

8c Rearrange $10x-5y+3=0$

$5y=10x+3$

$y=2x+\frac{3}{5}$

Gradient is 2 so perpendicular to $6x+12y=1$ since $2\times\left(-\frac{1}{2}\right)=-1$

9a Gradient of l_1 is 5

$y+3=5(x-3)$

$y+3=5x-15$

$5x-y-18=0$

9b Gradient of perpendicular is $-\frac{1}{5}$ since $-\frac{1}{5}\times 5=-1$

$y-1=-\frac{1}{5}(x+4)$

$-5y+5=x+4$

$x+5y-1=0$

10a Gradient of l_1 is $\frac{1}{2}$

$y-5=\frac{1}{2}(x+1)$

$2y-10=x+1$

$x-2y+11=0$

10b Gradient of perpendicular is -2 since $-2\times\frac{1}{2}=-1$

$y-2=-2(x-6)$

$y-2=-2x+12$

$2x+y-14=0$

11a Rearrange $l_1: y=9-3x$ so gradient of l_1 is -3

$$y+2=-3(x-8)$$
$$y+2=-3x+24$$
$$3x+y-22=0$$

11b Gradient of perpendicular is $\frac{1}{3}$ since $\frac{1}{3}\times(-3)=-1$

$$y+1=\frac{1}{3}(x+1)$$
$$3y+3=x+1$$
$$x-3y-2=0$$

12a Rearrange $l_1: 5y=-6x-2$

$$y=-\frac{6}{5}x-\frac{2}{5}$$

so gradient of l_1 is $-\frac{6}{5}$

$$y=-\frac{6}{5}(x-4)$$
$$5y=-6x+24$$
$$6x+5y-24=0$$

12b Gradient of perpendicular is $\frac{5}{6}$ since $\frac{5}{6}\times\left(-\frac{6}{5}\right)=-1$

$$y-3=\frac{5}{6}(x-12)$$
$$6y-18=5x-60$$
$$5x-6y-42=0$$

13a Rearrange $l_1: 2y=6x-1$

$$y=3x-\frac{1}{2}$$

so gradient of l_1 is 3

$$y-1=3\left(x-\frac{1}{2}\right)$$
$$y-1=3x-\frac{3}{2}$$
$$2y-2=6x-3$$
$$6x-2y-1=0$$

13b Gradient of perpendicular is $-\frac{1}{3}$ since $\left(-\frac{1}{3}\right)\times 3=-1$

$$y+\frac{1}{2}=-\frac{1}{3}(x+1)$$
$$6y+3=-2(x+1)$$
$$6y+3=-2x-2$$
$$2x+6y+5=0$$

14a Midpoint is $\left(\frac{5+-3}{2},\frac{-7+5}{2}\right)=(1,-1)$

Gradient is $\frac{5--7}{-3-5}=\frac{12}{-8}$

$$=-\frac{3}{2}$$

So gradient of perpendicular bisector is $m=\frac{2}{3}$ since $\frac{2}{3}\times\left(-\frac{3}{2}\right)=-1$

Equation is $y+1=\frac{2}{3}(x-1)$

$$2x-3y-5=0$$

14b Midpoint is $\left(\frac{-5+5}{2},\frac{-9+5}{2}\right)=(0,-2)$

Gradient is $\frac{5--9}{5--5}=\frac{14}{10}$

$$=\frac{7}{5}$$

So gradient of perpendicular bisector is $m=-\frac{5}{7}$ since $\left(-\frac{5}{7}\right)\times\frac{7}{5}=-1$

Equation is $y+2=-\frac{5}{7}x$

$$5x+7y+14=0$$

14c Midpoint is $\left(\frac{-6+4}{2},\frac{2+12}{2}\right)=(-1,7)$

Gradient is $\frac{12-2}{4--6}=\frac{10}{10}$

$$=1$$

So gradient of perpendicular bisector is $m=-1$ since $(-1)\times 1=-1$

Equation is $y-7=-(x+1)$

$$y=-x+6$$

14d Midpoint is $\left(\frac{2+-1}{2},\frac{-7+2}{2}\right)=\left(-\frac{1}{2},-\frac{5}{2}\right)$

Gradient is $\frac{2--7}{-1-2}=\frac{9}{-3}$

$$=-3$$

So gradient of perpendicular bisector is $m=\frac{1}{3}$ since $\frac{1}{3}\times(-3)=-1$

Equation is $y+\frac{5}{2}=\frac{1}{3}\left(x+\frac{1}{2}\right)$

$$x-3y-7=0$$